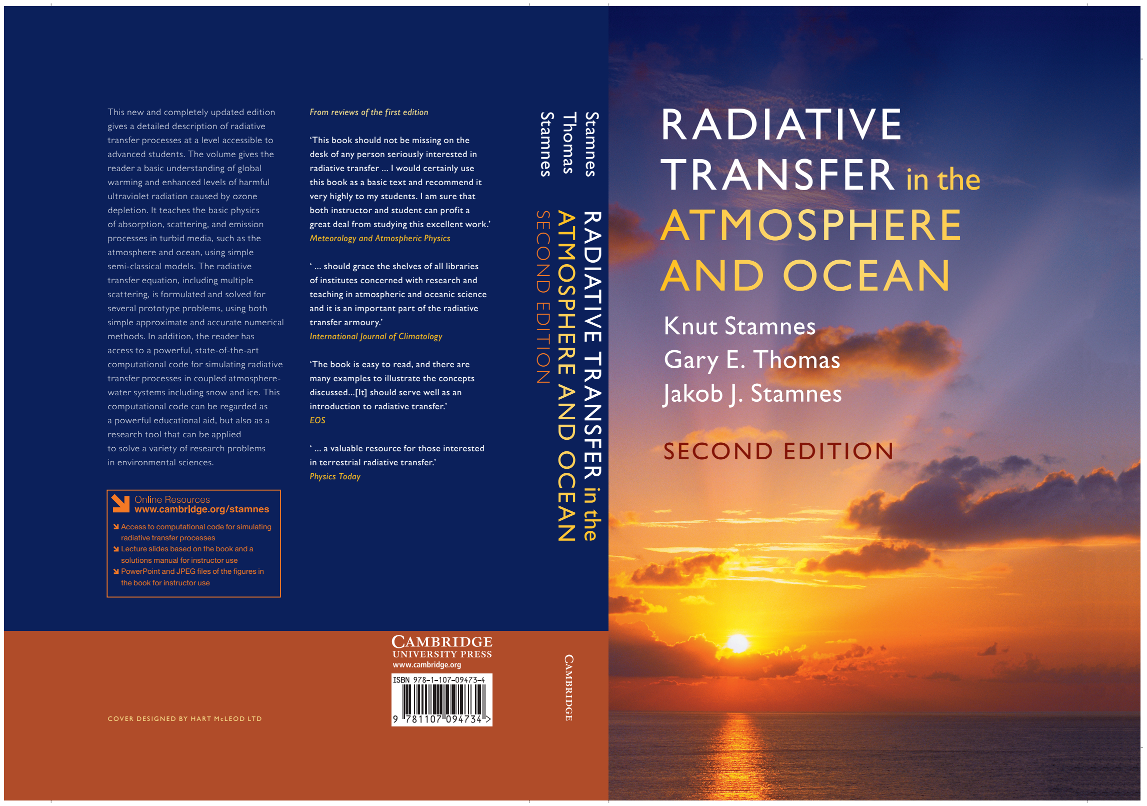
Energy Balance and Climate on Planet *Earth:* the Global Warming Problem



Based primarily on Chapter 8 in K. Stamnes, G. E. Thomas, and J. J. Stamnes, Radiative Transfer in the Atmosphere and Ocean, Cambridge University Press, 2017.

Recent Heat Wave: Human-induced or not?

Editorial in New York Times (07-11-2012):

The recent heat wave that has fried much of the country, ruined crops and led to heat-related deaths has again raised the question whether this and other extreme weather events can be attributed to human-induced climate change. The answer, increasingly, is a qualified yes.

- Mainstream scientists have always been cautious about drawing a causal link between global warming and any specific weather event....., but,....they agree that without sustained efforts to reduce greenhouse gases, dangerous heat waves are likely to become more common, as will prolonged droughts and coastal flooding.
- Many politicians and a vocal minority of scientists dispute such predictions as alarmist. But the numbers cannot be disputed:
  - **11 years from 2001-11 rank among the 13 warmest globally since record-keeping began 132 years ago;** (NOAA)
  - the **average temperature** in the contiguous United States **for the first six months of this year were the hottest recorded since 1895.** (NCDC)

According to IPCC: "It is virtually certain that increases in the frequency and magnitude of warm daily temperature extremes and decreases in cold extremes will occur in the 21st century on the global scale."

Earth's Energy Budget and Climate: Goal

The goal is to explain in basic physical terms how the temperature profile of the Earth (shown below) comes about, and how it is determined by the **Greenhouse Effect**.

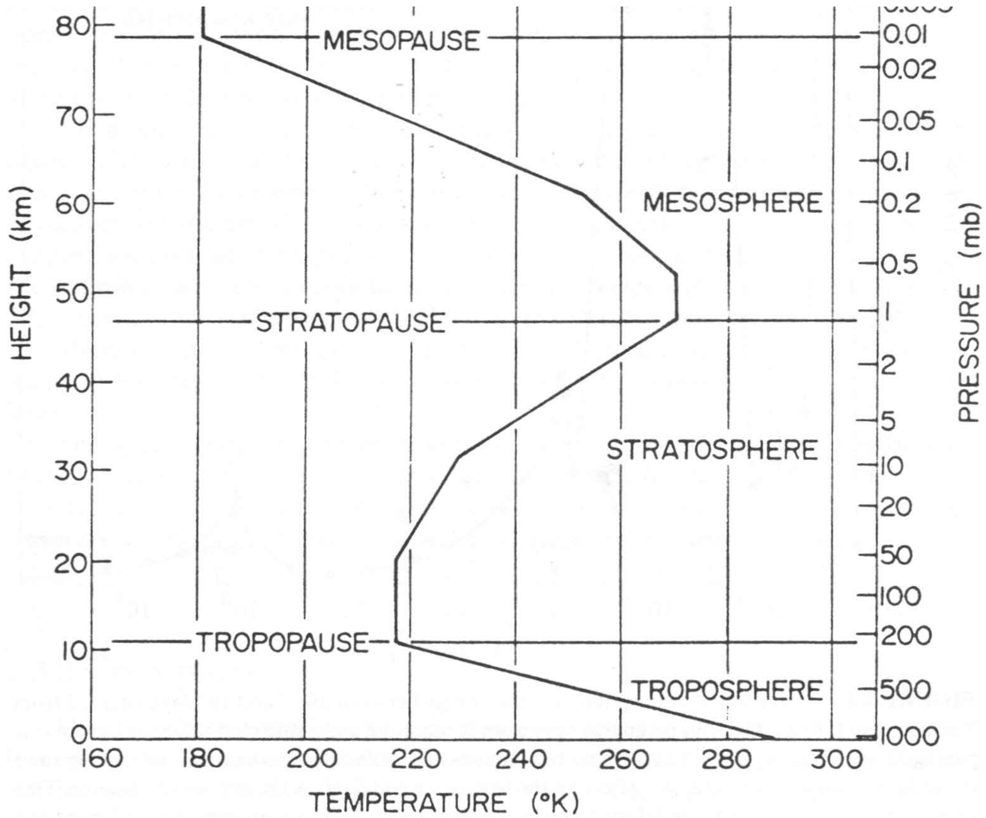


Figure 1: **The 1976 Standard Atmospheric Temperature Profile.**

Earth Energy Budget and Climate: Introduction (1)

The main purpose of this lecture is to explain **the greenhouse effect**, and discuss the **global warming** issue from a basic physical point of view. To this end we shall discuss:

- The temperature of an airless planet (no atmosphere).
- The basic **greenhouse effect** resulting from an atmosphere that is transparent to solar radiation, but traps infrared (IR) radiation.
- The impact of clouds and aerosols on the energy budget and climate.
- The impact of absorption of solar radiation: the **anti-greenhouse** effect.
- The impact of increased levels of carbon dioxide and other greenhouse gases.

To facilitate this discussion, we will start by reviewing some basic physical laws and concepts, and then use simple models to get the basic ideas across. In particular, we will show that the basic features of Earth's climate:

- can be captured in simple algebraic expressions that will allow anybody equipped with basic knowledge of mathematics to explore how:
- changes in **cloud cover, aerosol loading, surface albedo, and greenhouse gas concentrations** impact the climate of the Earth.

Earth Energy Budget and Climate: Introduction (2)

Definitions/Explanations:

Light (or radiation) interacts with matter through **absorption** and **scattering**. For a planetary system like the Earth's **atmosphere-surface** system:

albedo = fraction of energy reflected by the planetary atmosphere or surface
σ = scattering cross section per particle in the planet's atmosphere
α = absorption cross section per particle in the planet's atmosphere
$\tau = (\sigma + \alpha) n \cdot s = (\sigma + \alpha)\mathcal{N}$ = **optical depth** or **opacity**, where
n = the concentration of absorbing/scattering particles per unit volume
s = the distance along the light beam
$\mathcal{N} = n\, s$ = column number per unit area.
irradiance = energy per unit area per unit time.

Acronyms:

NOAA: National Oceanic and Atmospheric Administration
NCDC: National Climate Data Center
IPCC: Intergovernmental Panel on Climate Change

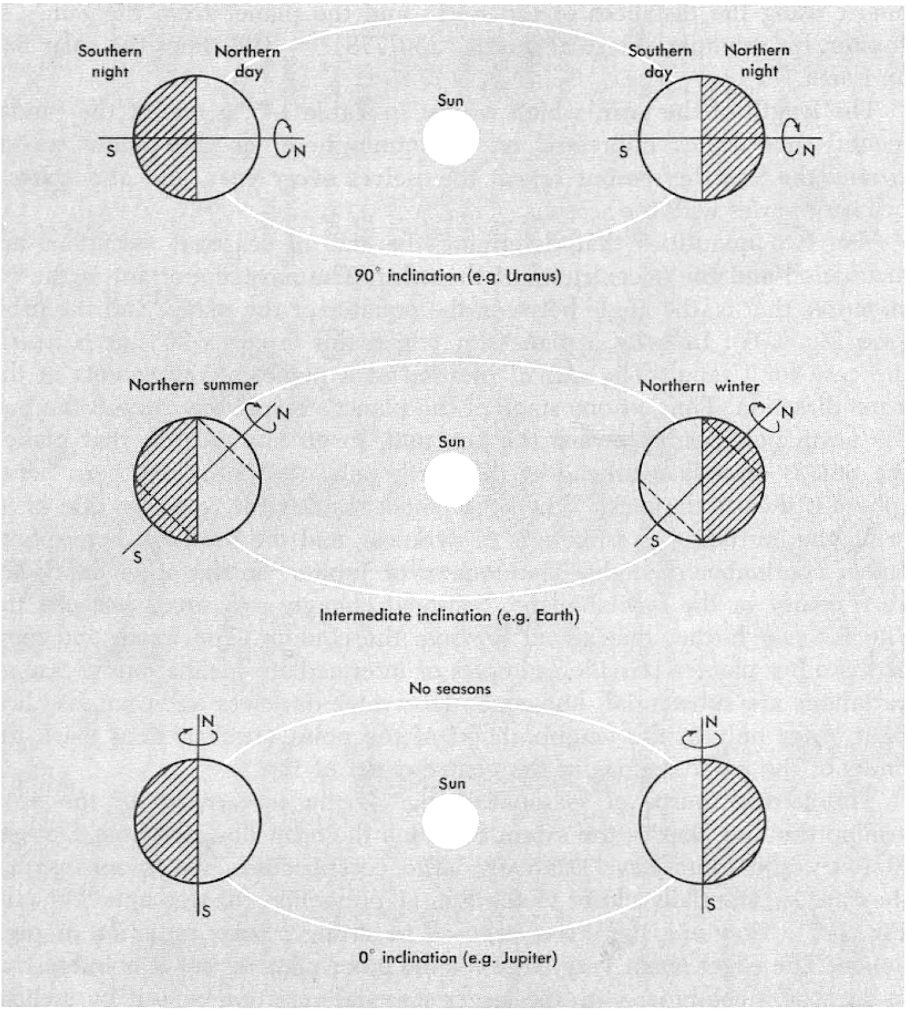


Figure 2: **How seasonal variations depend on the angle between the equator and the plane of the orbit. If a planet had exactly 90° inclinations, it would be impossible to draw an analogy with terrestrial north and south poles. The labels in the top panel would then be arbitrary.**

Earth Energy Budget and Climate: Introduction (4)

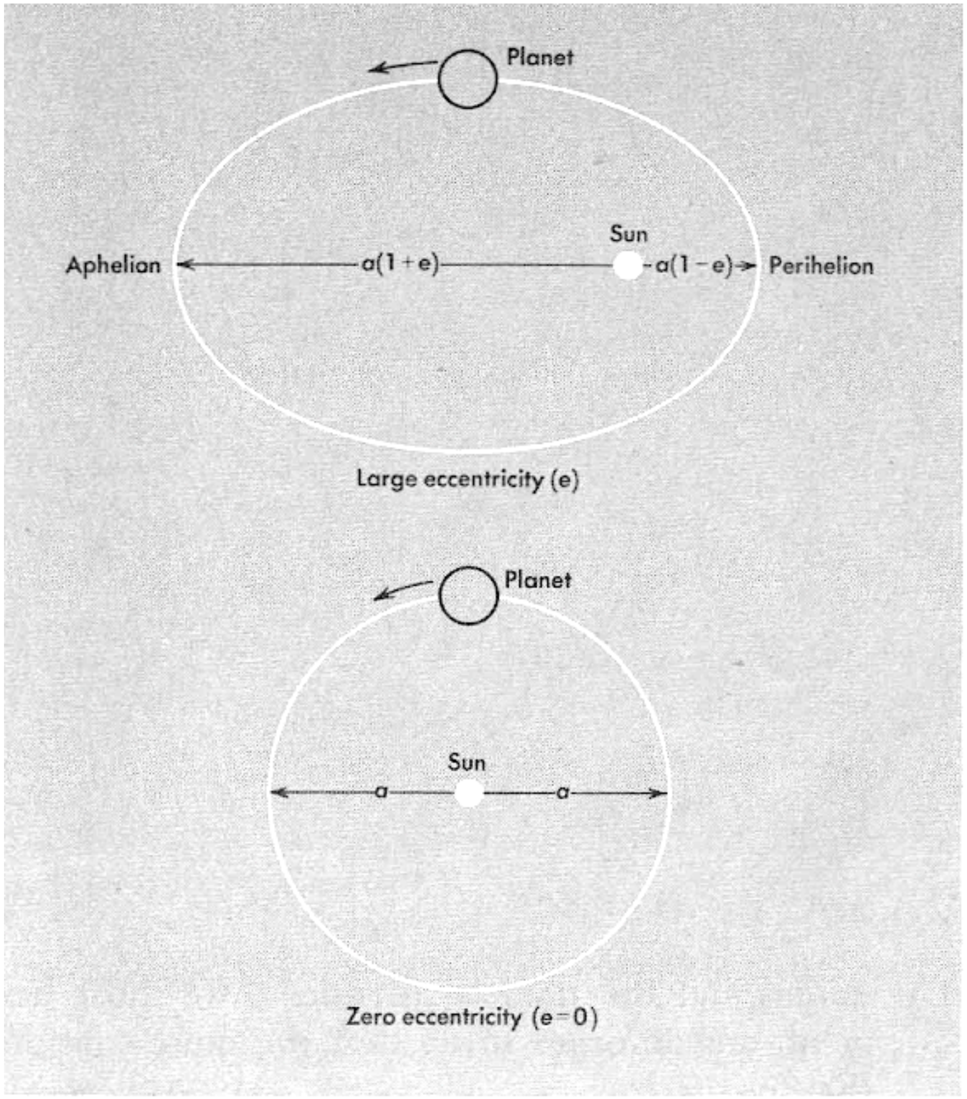


Figure 3: **How orbital eccentricity causes seasonal changes in the distance from the Sun.**

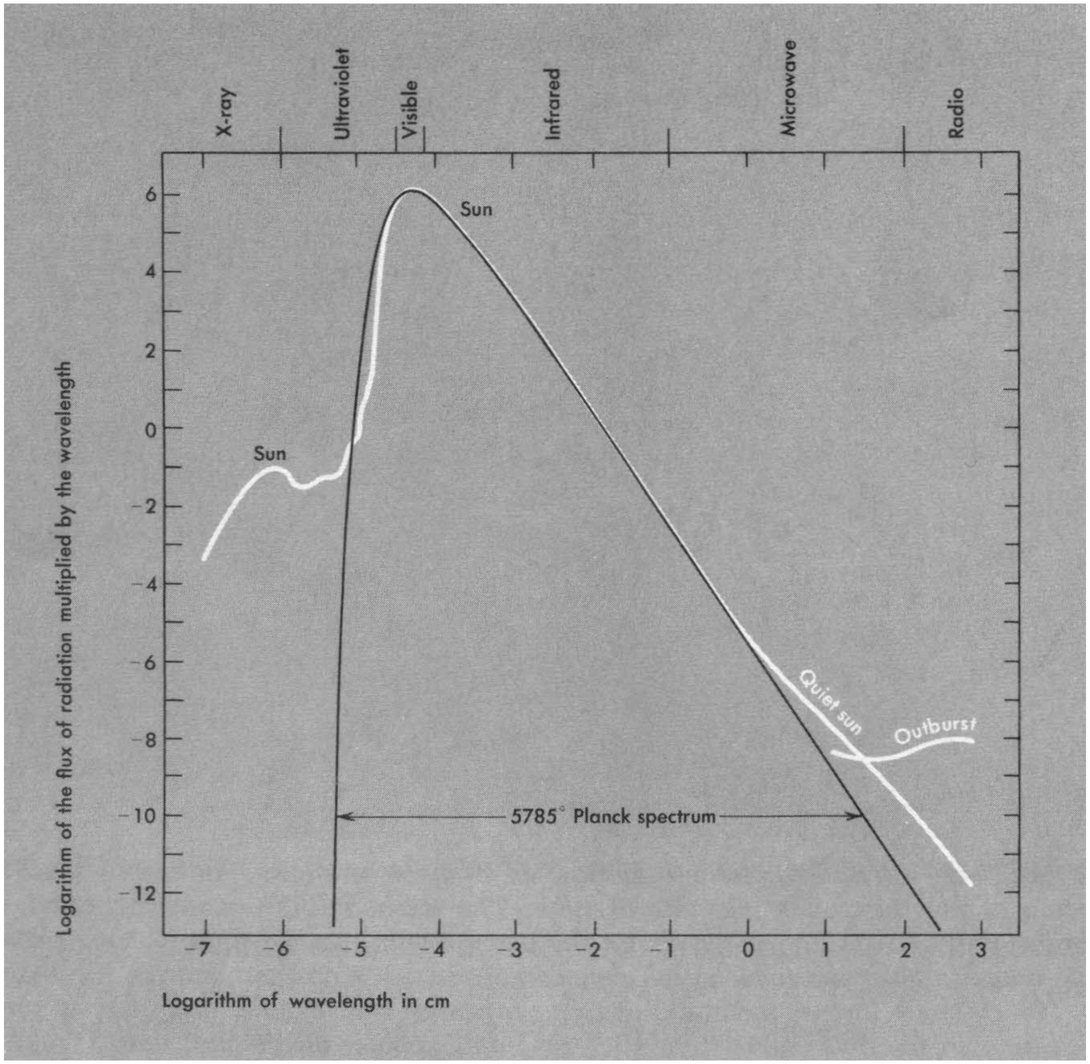


Figure 4: **Comparison between the spectrum of the Sun and a blackbody (Planck) spectrum corresponding to a temperature of 5,785 K.**

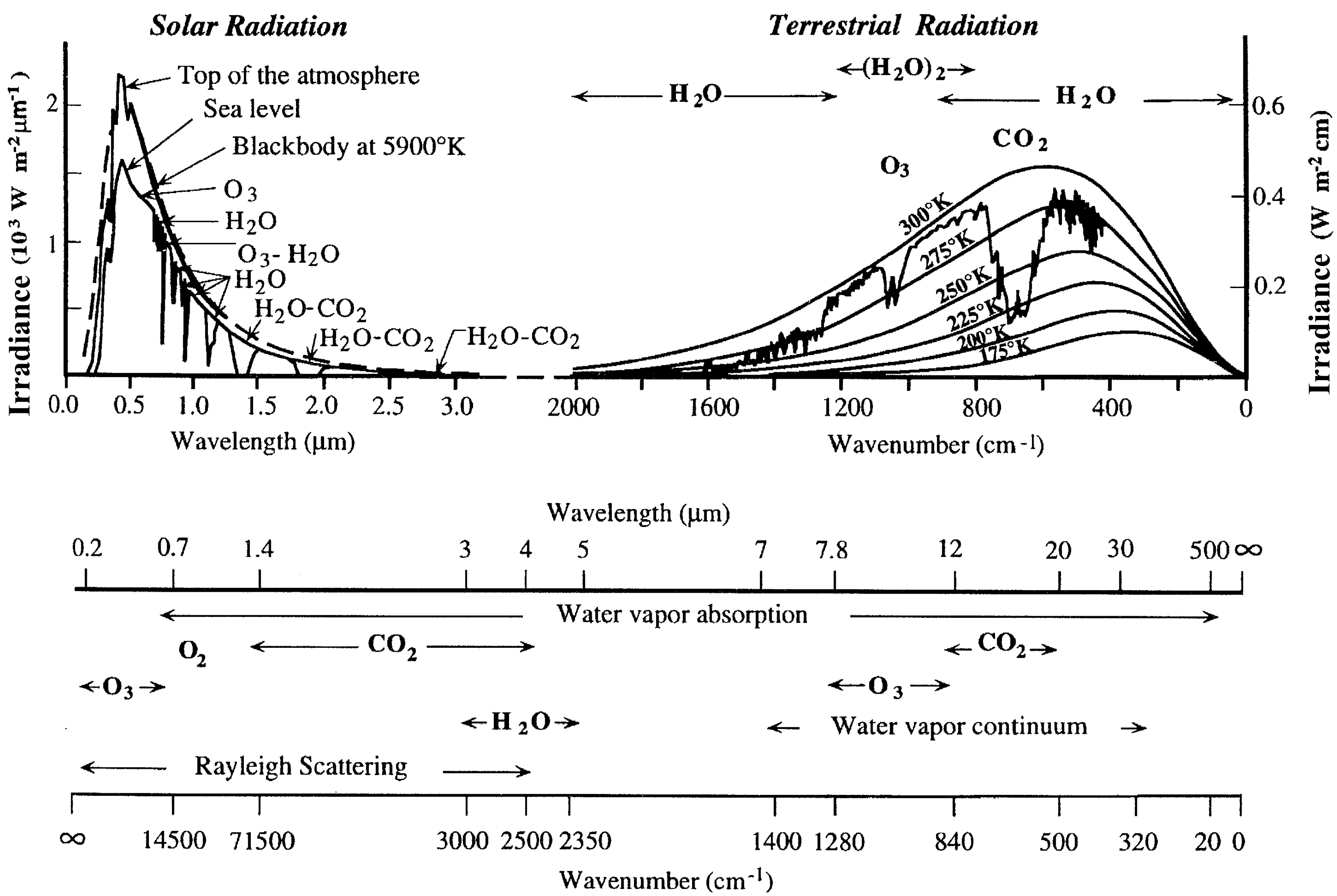


Figure 5: **Spectral distribution of solar and terrestrial radiation fields. Also shown are the approximate shapes and positions of the scattering and absorption features of the Earth's atmosphere.**

Earth Energy Budget and Climate: Solar Spectrum

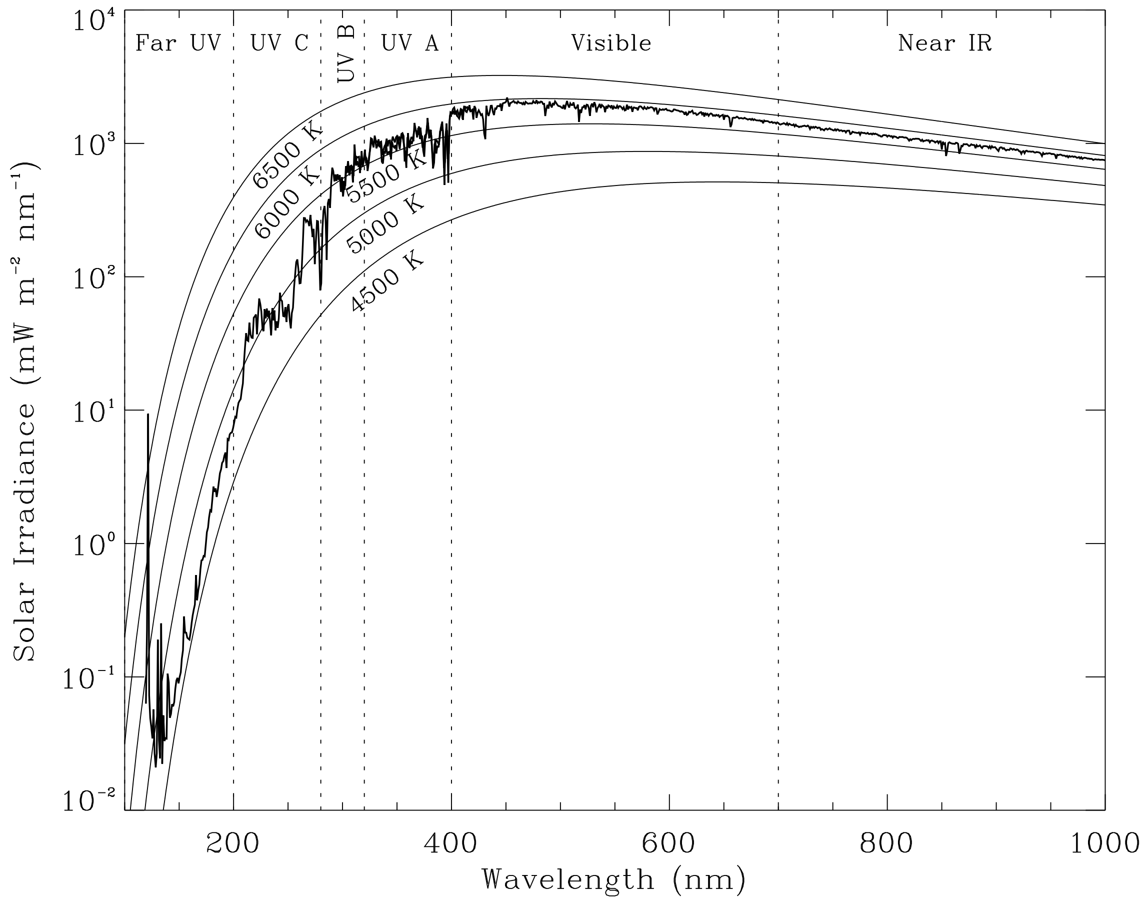


Figure 6: **Extraterrestrial solar irradiance, measured by a spectrometer on board an Earth-orbiting satellite. The UV spectrum ($119 < \lambda < 420$ nm) was measured by the SOLSTICE instrument on the UARS satellite. The vertical lines divide the various spectral sub-ranges. The smooth curves are calculated blackbody spectra for a number of temperatures.**

Earth's Energy Budget and Climate: Temperature

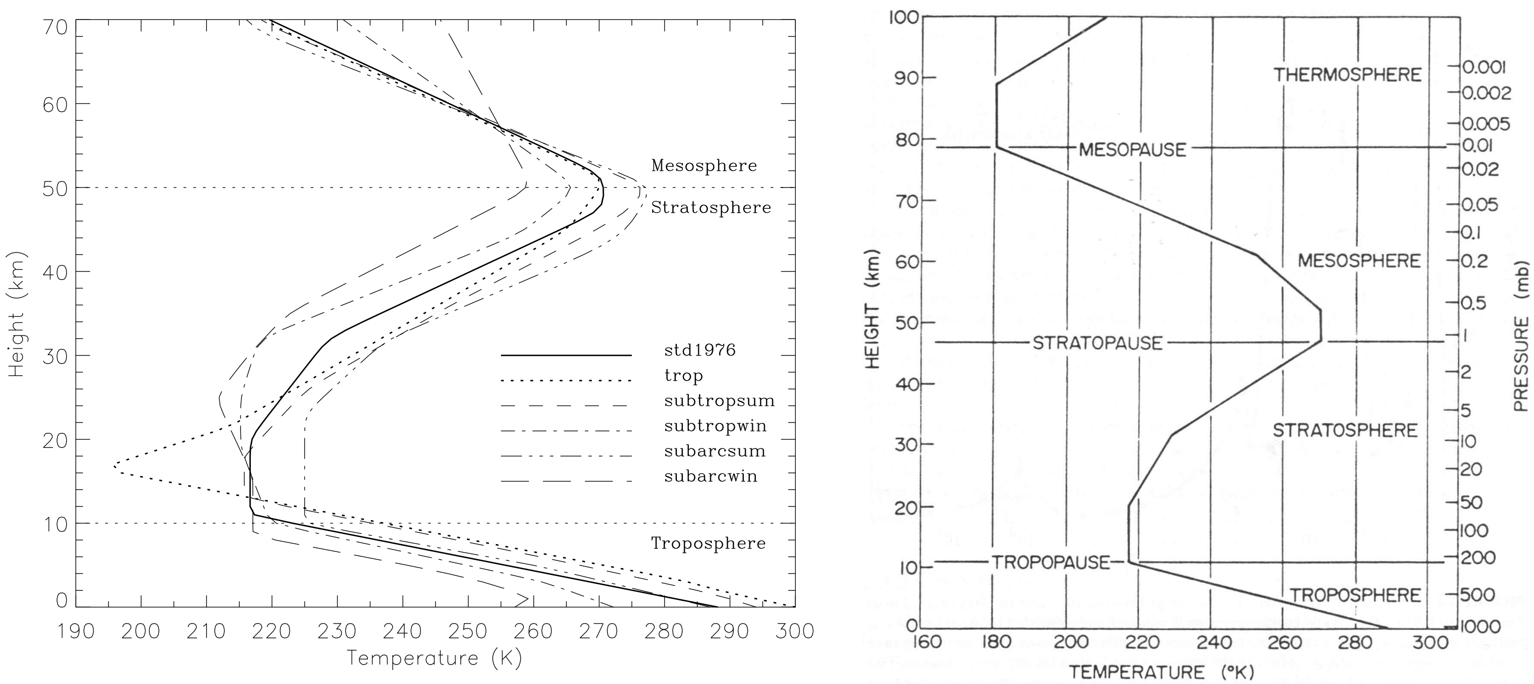


Figure 7: **Left: Standard empirical model temperature profiles for various locations and seasons. The 1976 Standard atmosphere is appropriate for the global mean. The Tropical atmosphere is valid for latitudes less than 30°; the Subtropical model for 30–45°; the Subarctic atmosphere for 45–60°; and the Arctic for 60–90°. Right: The 1976 Standard Atmospheric Temperature Profile.**

Earth Energy Budget and Climate: Basic Physics (1)

The **equilibrium radiation field**, expressed in the famous **Radiation Laws**, first established by Kirchhoff in 1860, depends only upon:

- the **temperature** of the **medium** and is **totally independent of the nature of the matter** of which the **medium** consists.

Planck derived the hemispherical blackbody spectral irradiance $F_\nu^{\rm BB}$:

$$F_\nu^{\rm BB} = \frac{m_{\rm r}^2}{c^2} \frac{2\pi h\nu^3}{\left(e^{h\nu/k_{\rm B}T} - 1\right)} \qquad \longleftarrow \qquad \textbf{Planck's radiation law} \qquad (1)$$

where h is Planck's constant, $m_{\rm r}$ is the real part of the refractive index, and $k_{\rm B}$ is Boltzmann's constant. The blackbody radiation field is:

- isotropic and unpolarized, and the net irradiance is everywhere zero. Therefore,

the blackbody radiance is related to the hemispherical irradiance through $F_\nu^{\rm BB} = \int_{2\pi} d\omega \cos\theta I_\nu^{\rm BB} = \pi I_\nu^{\rm BB}$:

$$\boxed{I_\nu^{\rm BB} = B_\nu(T) \equiv \frac{m_{\rm r}^2}{c^2} \frac{2h\nu^3}{\left(e^{h\nu/k_{\rm B}T} - 1\right)} \qquad \longleftarrow \qquad \textbf{Planck function}. \qquad (2)}$$

- The closely related function $B_\lambda(T)$ is illustrated in Fig. 8 for 3 temperatures.

Earth's Energy Budget and Climate: Basic Physics (2)

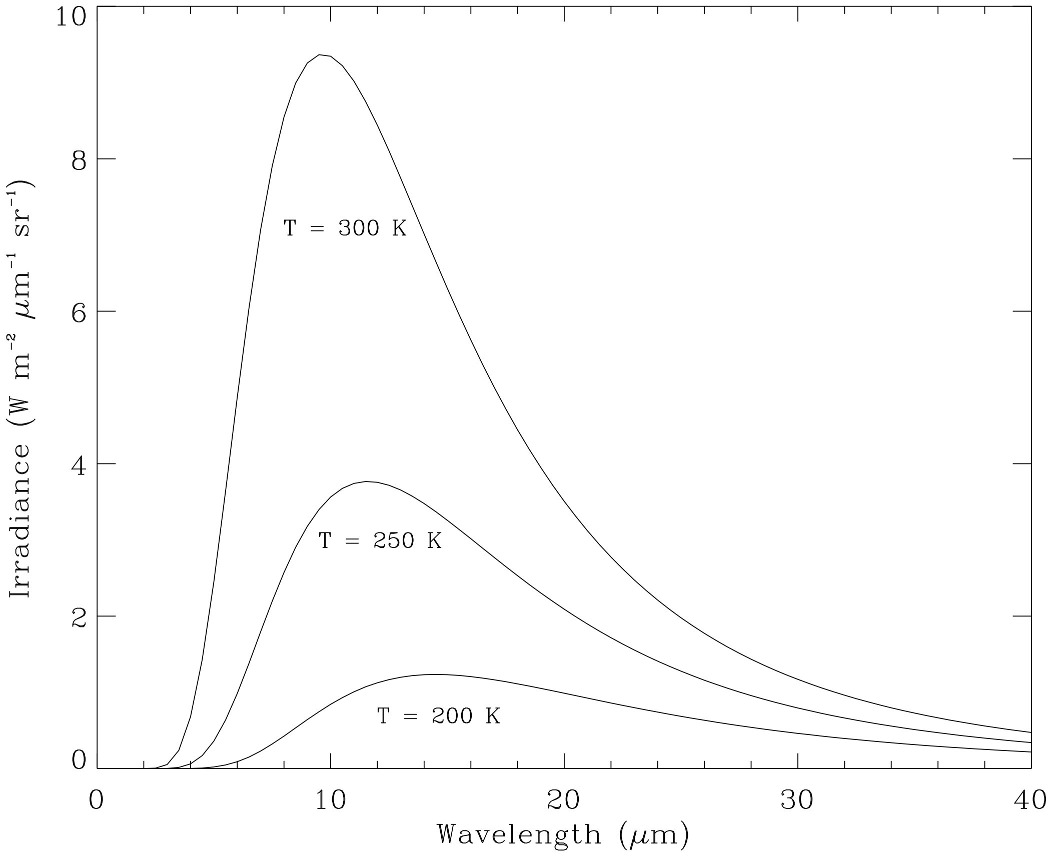


Figure 8: **The blackbody radiance B_λ versus wavelength λ. The relationship between B_ν and B_λ is $B_\lambda|d\lambda| = B_\nu|d\nu|$. Since $\lambda = c/\nu$ then $|d\nu| = (c/\lambda^2)|d\lambda| \quad \Rightarrow \quad B_\lambda = \frac{1}{c^2\lambda^5}\frac{2h}{\left(e^{hc/k_{\rm B}\lambda T}-1\right)}$.**

Earth Energy Budget and Climate: Basic Physics (3)

The frequency-integrated hemispherical irradiance leaving the medium is:

$$F^{\rm BB} = \int_0^\infty d\nu \int_{2\pi} d\omega \cos\theta I_\nu^{\rm BB} = \pi \int_0^\infty d\nu B_\nu(T). \tag{3}$$

Substituting Eq. 2 in Eq. 3, we obtain:

$$\boxed{F^{\rm BB} = \sigma_{\rm B} T^4 \qquad \longleftarrow \qquad \textbf{Stefan-Boltzmann law} \tag{4}}$$

$$\sigma_{\rm B} = 2\pi^5 k_{\rm B}^4 / 15 h^3 c^2 = 5.6703 \times 10^{-8}\ [\mathrm{W \cdot m^{-2} \cdot K^{-4}}].$$

- According to the First Law of Thermodynamics, the time rate of change of the globally averaged column energy $\bar{E}$ is given by ($\bar{\rho}$ is the spherical albedo*):

$$\frac{\partial \bar{E}}{\partial t} \equiv \bar{N} = \overbrace{(1-\bar{\rho})\bar{F}^{\rm s}}^{\text{incident energy}} - \overbrace{\bar{F}_{\rm TOA}}^{\text{emitted energy}} \qquad \longleftarrow \qquad \textbf{radiative forcing} \tag{5}$$

where $\bar{N}$ is the mean **radiative forcing**, i.e., to the net irradiance at the top of the atmosphere (TOA).

  - $\bar{F}^{\rm s}$ is the mean solar irradiance falling on the planet,
  - $(1-\bar{\rho})\bar{F}^{\rm s}$ is the fraction of the mean solar irradiance absorbed by the planet,
  - $\bar{F}_{\rm TOA}$ is the mean thermal IR emission at the TOA.

*albedo = fraction of incident energy that is reflected.

Earth's Energy Budget and Climate: Basic Physics (4)

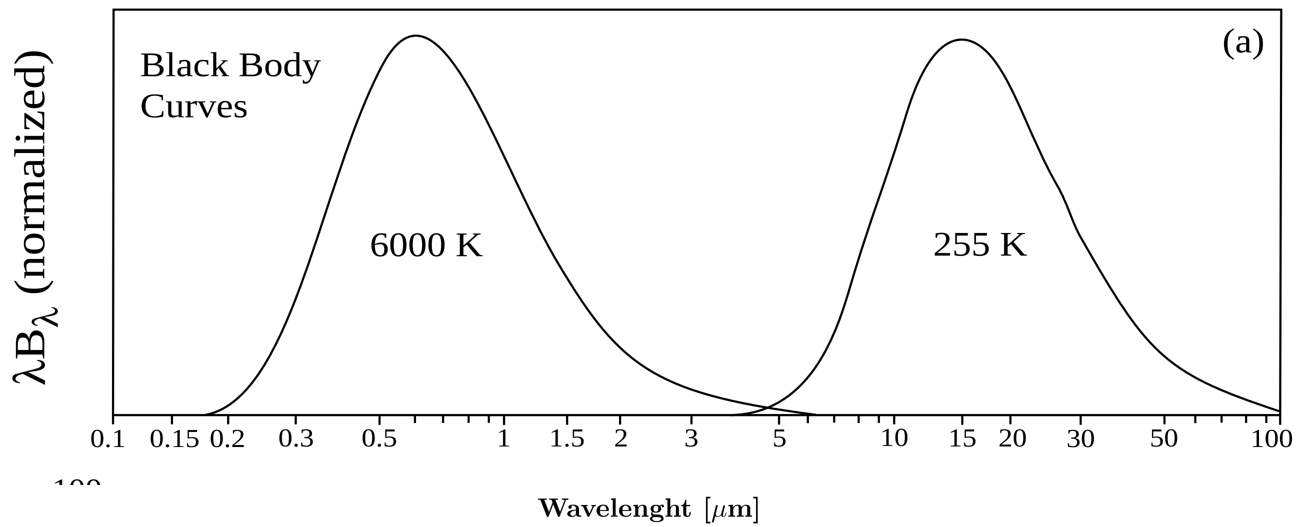


Figure 9: **Blackbody curves for radiation from the Sun (6000 K) and the Earth (255 K).**

The area under each of the blackbody curves is given by:

$$F^{\mathrm{BB}} = \sigma_{\mathrm{B}} T^4 \qquad \longleftarrow \qquad \textbf{Stefan-Boltzmann law}.$$

Note that there is very little overlap between the solar and the terrestrial (Earth) radiation spectra. Thus:

- **the infrared (IR) or Earth radiation ($\lambda > 3.5\ \mu$m) is spectrally separated from the Solar (or visible) ($\lambda < 3.5\ \mu$m) radiation.**

Earth Energy Budget and Climate: Basic Physics (5)

Characterizing outgoing IR radiation by an effective temperature T_e, we obtain

$$\bar{F}_\mathrm{TOA} = \sigma_\mathrm{B} T_\mathrm{e}^4 \quad \longleftarrow \quad \textbf{energy emitted by a planet.}$$

- By averaging over one or several years, we find a time- and space-averaged radiative forcing, which is close to zero, i.e., $\langle \bar{N} \rangle \approx 0$.
- When $\langle \bar{N} \rangle = 0$, we have **planetary radiative equilibrium (RE)**, implying:

$$\langle \bar{N} \rangle = (1-\bar{\rho})\bar{F}^\mathrm{s} - \bar{F}_\mathrm{TOA} = \overbrace{(1-\bar{\rho})\bar{F}^\mathrm{s}}^{\text{incident energy}} - \overbrace{\sigma_\mathrm{B} T_\mathrm{e}^4}^{\text{emitted energy}} = 0.$$

Solving for T_e, the effective temperature of the planet becomes:

$$\boxed{T_\mathrm{e} = \left[\frac{(1-\bar{\rho})\bar{F}^\mathrm{s}}{\sigma_\mathrm{B}}\right]^{1/4} \quad \longleftarrow \quad \textbf{planetary RE temperature}. \qquad (6)}$$

- For Earth the spherical albedo is $\bar{\rho} = 0.3$, and $\bar{F}^\mathrm{s} = S_0/4 = 342$ W·m^{-2}.[†]
- The above formula yields $T_\mathrm{e} = -18$ °C, **much lower than Earth's mean surface temperature** $T_\mathrm{s} = +15$ °C, due to the **greenhouse effect**.

[†]Here $S_0 = 1368$ W·m^{-2} = the solar "constant", and the factor of $1/4$ is the ratio of disk area of the Earth (πR^2) intercepting the radiation to the the total surface area ($4\pi R^2$).

The Role of Radiation in Climate (1)

Several radiatively-active molecules – the so-called **greenhouse gases**:

- strongly absorb and emit infrared (IR) radiation, and thereby trap radiative energy that would otherwise escape to space.

The **global warming** issue is concerned with the effects of:

- enhanced abundances of these **greenhouse gases** and changes in **aerosol loading** (due in part to man's activities) and **cloud cover** on the overall radiative energy balance of the Earth and hence on climate.
- The bulk of the Earth's atmosphere (99% by mass) consists of molecular nitrogen (N_2) and oxygen (O_2) $\Longrightarrow$ **radiatively insignificant, homonuclear, diatomic molecules.**
- **Trace amounts of polyatomic molecules (CO_2, H_2O, O_3, CH_4)** are responsible for atmospheric absorption and emission of radiation.

Below is a schematic diagram of the significant components of the Earth's energy balance:

- The incoming solar irradiance is 342 W·m^{-2} averaged over the entire planet.

The Role of Radiation in Climate (2)

- Of this irradiance: 107 W·m^{-2} (about 31%) is reflected to space; whereas
- the absorbed solar energy, 235 W·m^{-2}, is balanced by an equal amount radiated to space in the IR.

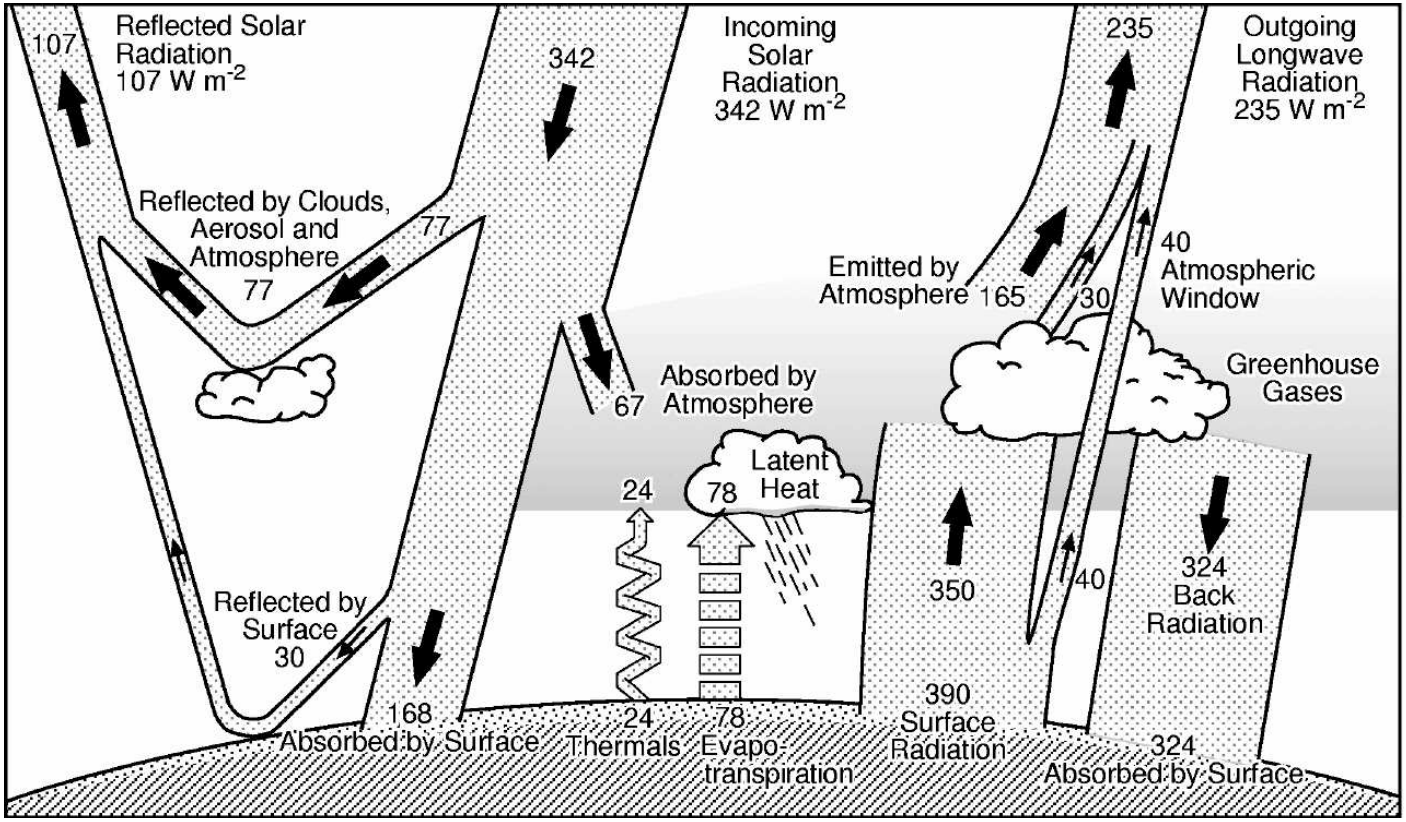


Figure 10: **Earth's energy budget.**

The Role of Radiation in Climate (3)

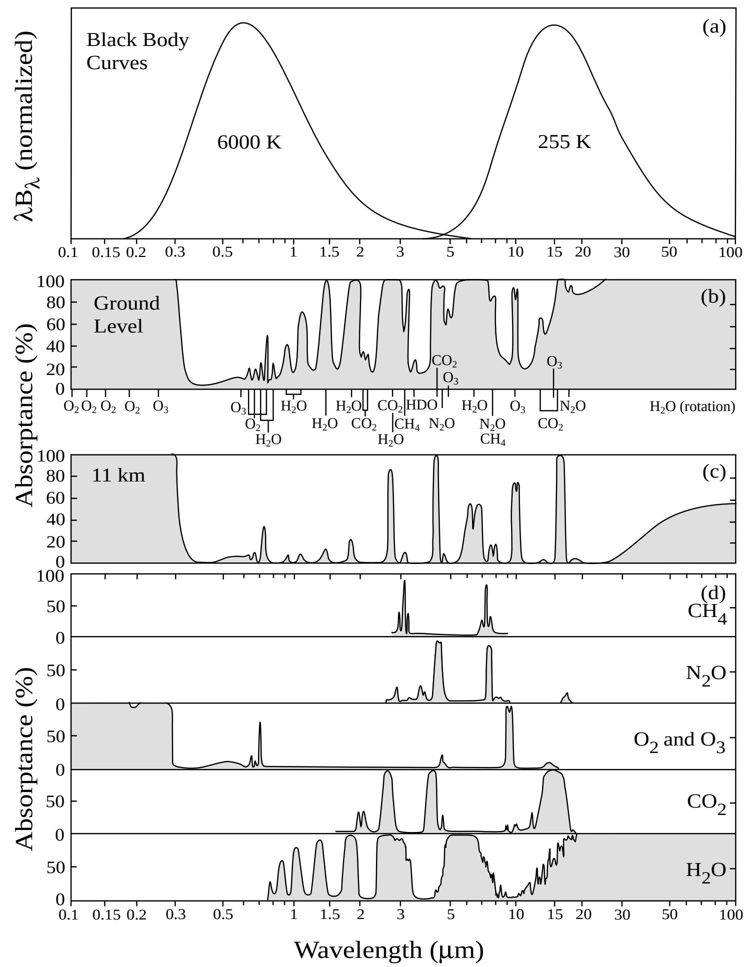


Figure 11: **(a) Blackbody curves for solar radiation (6000 K) and terrestrial radiation (255 K). Absorption spectra for (b) the entire vertical extent of the atmosphere, (c) the portion of the atmosphere above 11 km, and (d) for the various atmospheric gases between the top and the surface of the Earth.**

The Role of Radiation in Climate (4)

Within the atmosphere, the land surface and the ocean:

- the transformation of radiative energy into chemical, thermal and kinetic energy drives the "engine" of weather and climate. BUT:

The most important radiative interaction is the **greenhouse effect:**

- **Earth would be in a state of permanent glaciation** without it.

But enhanced levels of CO_2 abundance (and other GHGs), is a concern, because:

- they absorb and trap Earth radiation that would otherwise escape to space, and
- they cause an imbalance between the energy received and emitted by Earth. If Earth receives more energy from the Sun than it is able to emit to space, then:
- by increasing its effective temperature it will increase the energy emitted (by the **Stefan-Boltzmann law:** $F^{\mathrm{BB}} = \sigma_{\mathrm{B}} T^4$) until a new radiative equilibrium between the Sun and the Earth is established.

Similary, the trapping of terrestrial radiation by enhanced levels of GHGs:

- will lead to a **surface warming**, which can be explained in terms of an increase in the emission height, so that the net energy emitted by the Earth is equal to that received from the Sun.

The Role of Radiation in Climate (5)

We will consider how both visible and IR radiative interactions affect the Earth's climate. We will **ignore**:

- many other climate variables (precipitation, wind, ice and snow cover, etc.), and:
- deal only with the temperature structure (the global warming problem).

In order to isolate the basic radiative forcing, and the atmosphere's most elementary response, we employ an analytic model:

- based on a simple solution to the radiative transfer equation, because:
- this model is remarkably flexible, and will describe the static, **globally-averaged temperature structure** of a planet's atmosphere.

The ocean serves as a heat storage medium, and in this simple picture:

- it acts only to delay the approach of the climate system to an equilibrium state.

In more realistic regional models the main role of the ocean is to:

- mediate seasonal responses, and transport heat poleward from the low-latitude regions of heating excess.

Role of Radiation in Climate: Radiative Equilibrium (1)

Radiative equilibrium (RE) is based on the assumption that the **atmosphere has negligible absorption of visible (solar) radiation**. The surface is assumed to be reflective in the visible, but black in the IR. Thus:

- **the surface is heated by incoming solar radiation, and by downward IR radiation from the atmosphere;**
- at the upper 'edge' where the slant optical depth is of order unity, **the atmosphere radiates to space with a globally-averaged effective temperature** T_e determined by the overall energy balance.

As already shown, the effective temperature of an **airless** planet is given by:

$$T_e = \left[\frac{S_0(1-\bar{\rho})}{4\sigma_B}\right]^{1/4} \qquad \text{where} \tag{7}$$

- S_0 = "solar constant"; $\bar{\rho}$ = spherical albedo; σ_B = Stefan-Boltzmann constant.

With no 'blanketing' atmosphere to trap radiation emitted by the surface:

- the effective temperature is equal to the surface temperature. For the Earth, the effective temperature is $T_e = 255$ K or -18°C $\longleftarrow$ **too cold!!**

Role of Radiation in Climate: Radiative Equilibrium (2)

Assumptions:

1. IR radiation interacts with a single gray absorber with absorption coefficient α;
2. no scattering occurs (the scattering coefficient $\sigma = 0$);
3. the atmospheric optical properties vary only in the vertical direction;
4. IR ($\lambda > 3.5\ \mu$m) spectrally separated from visible ($\lambda < 3.5\ \mu$m) radiation.

$$B(\tau) = \int_0^\infty d\nu B_\nu[T(\tau)] = \frac{\sigma_\mathrm{B} T^4(\tau)}{\pi} \quad \longleftarrow \quad \textbf{gray approximation} \tag{8}$$

where the IR optical depth is the vertical position variable: $\tau(z) = \int_z^\infty dz \alpha(z)$.

We need to solve ($u = \cos\theta$, where θ is the polar angle):

$$u \frac{dI(\tau, u)}{d\tau} = I(\tau, u) - B(\tau) \quad \longleftarrow \quad \textbf{radiative transfer equation}. \tag{9}$$

Integrating Eq. 9 over all solid angles, we obtain:

$$\frac{dF(\tau)}{d\tau} = 4\pi[\bar{I}(\tau) - B(\tau)]. \tag{10}$$

In Eq. 10, $F(\tau)$ is the net irradiance, and $\bar{I}(\tau)$ is the mean radiance ($\mu \equiv |u|$):

$$F(\tau) = 2\pi \int_{-1}^{1} du u I(\tau, u) = 2\pi\left[\int_0^1 d\mu \mu I^+(\tau, \mu) - \int_0^1 d\mu \mu I^-(\tau, \mu)\right] \qquad (11)$$

$$\bar{I}(\tau) = \frac{1}{2}\int_{-1}^{1} du\, I(\tau, u) = \frac{1}{2}\left[\int_0^1 d\mu I^+(\tau, \mu) + \int_0^1 d\mu I^-(\tau, \mu)\right] \qquad (12)$$

and $I^{\pm}(\tau, \mu)$ denote the radiances into the upper (+) and lower (−) hemispheres.

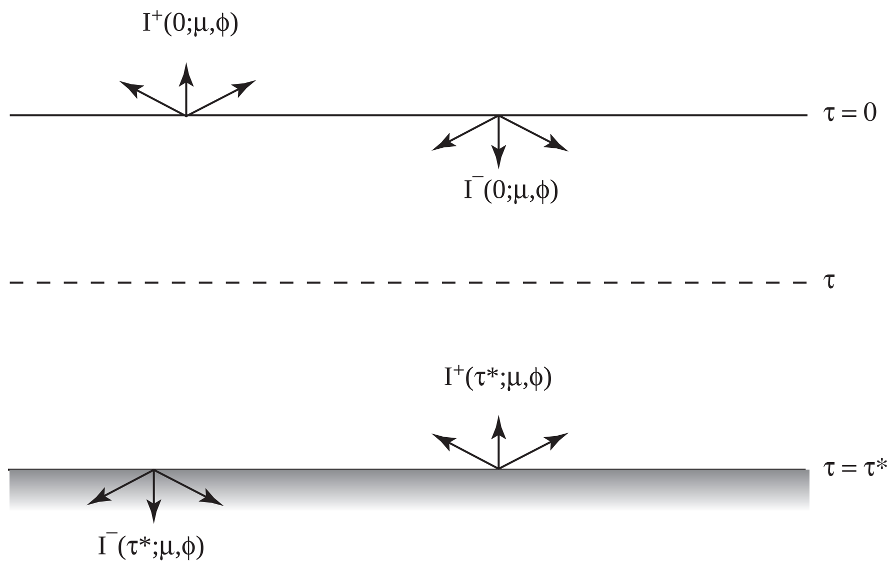


Figure 12: **Illustration of upward $I^+(\tau, \mu)$ and downward radiances $I^-(\tau, \mu)$ in slab geometry. The optical depth τ is measured downward from the "top" of the atmosphere ($\tau = 0$), to the surface ($\tau = \tau^*$). $\mu = |u| = |\cos\theta|$, and θ is the polar angle. The vertical height z is measured upward from $z = 0$ at the surface.**

Role of Radiation in Climate: Radiative Equilibrium (4)

The fifth assumption is that of **radiative equilibrium (RE):**

- 5. The absorption rate of IR radiation $[\alpha \bar{I}(\tau)]$ is equal to the emission rate $[\alpha B(\tau) = \alpha\, \sigma_{\rm B} T^4(\tau)/\pi]$: $\rightarrow \bar{I}(\tau) = B(\tau) = \sigma_{\rm B} T^4/\pi$.
- From Eq. 10, since $\bar{I}(\tau) = B(\tau)$:
  $dF(\tau)/d\tau = 0 \rightarrow F(\tau) = \sigma_{\rm B} T_{\rm e}^4 \;$ = constant in RE.

A final assumption is needed to obtain an analytic solution to Eq. 9:

$$u \frac{dI(\tau,u)}{d\tau} = I(\tau,u) - B(\tau) = I(\tau,u) - \overbrace{\frac{1}{2}\int_{-1}^{1} du\; I(\tau,u)}^{\bar{I}(\tau)} \tag{13}$$

- 6. The radiance varies slowly with $u = \cos\theta$: use **two-stream approximation (TSA)** to replace the entire angular distribution with $I^+(\tau)$ and $I^-(\tau)$:

$$I(\tau,u) = I^+(\tau) \;\text{ if }\; u > 0 \;\text{ and }\; I^-(\tau) \;\text{ if }\; u < 0 \tag{14}$$

$$F(\tau) \approx 2\pi\bar{\mu}[I^+(\tau) - I^-(\tau)] \quad \text{and} \quad \bar{I}(\tau) \approx \frac{1}{2}[I^+(\tau) + I^-(\tau)] \tag{15}$$

where we have used Eqs. 11 and 12, and $\bar{\mu}$ is the absolute value of the mean cosine of the rays, which lies between $1/2$ and $1/\sqrt{3}$. With these assumptions we can rewrite Eq. 13 as:

$$+\bar{\mu}\frac{dI^+(\tau)}{d\tau} = I^+(\tau) - \frac{1}{2}[I^+(\tau) + I^-(\tau)] = +\frac{1}{2}[I^+(\tau) - I^-(\tau)] \quad (16)$$

$$-\bar{\mu}\frac{dI^-(\tau)}{d\tau} = I^-(\tau) - \frac{1}{2}[I^+(\tau) + I^-(\tau)] = -\frac{1}{2}[I^+(\tau) - I^-(\tau)]. \quad (17)$$

Subtracting Eq. 17 from 16 yields:

$$\bar{\mu}\frac{d[I^+(\tau) + I^-(\tau)]}{d\tau} = I^+(\tau) - I^-(\tau) \quad (18)$$

but since $B(\tau) = (1/2)[I^+(\tau) + I^-(\tau)]$ and $I^+(\tau) - I^-(\tau) = \sigma_{\rm B} T_{\rm e}^4/\pi$, we find

$$\frac{dB(\tau)}{d\tau} = \frac{\sigma_{\rm B} T_{\rm e}^4}{2\pi\bar{\mu}} \quad \Longrightarrow \quad B(\tau) = \frac{\sigma_{\rm B} T_{\rm e}^4}{2\pi\bar{\mu}}\tau + C. \quad (19)$$

Since the IR radiation escapes the medium from an optical depth of unity, where $\tau/\bar{\mu} = 1$, and $T(\tau = \tau_{\rm e} = \bar{\mu}) = T_{\rm e}$, we have $B(\tau = \bar{\mu}) = \frac{\sigma_{\rm B} T_{\rm e}^4}{2\pi\bar{\mu}}\bar{\mu} + C = \sigma_{\rm B} T_{\rm e}^4/\pi$. Solving for C, we find $C = \sigma_{\rm B} T_{\rm e}^4/\pi - \frac{\sigma_{\rm B} T_{\rm e}^4}{2\pi\bar{\mu}}\bar{\mu} = \sigma_{\rm B} T_{\rm e}^4/2\pi$, and thus:

$$\boxed{B(\tau) = \frac{\sigma_{\rm B} T_{\rm e}^4}{2\pi}(1 + \tau/\bar{\mu}) \quad \longleftarrow \quad \textbf{radiative equilibrium}.} \quad (20)$$

Role of Radiation in Climate: Radiative Equilibrium (6)

Since $B(\tau) = \sigma_{\rm B} T^4(\tau)/\pi \equiv \sigma_{\rm B} T^4_{\rm re}(\tau)/\pi$ (Eq. 8), the RE temperature profile becomes:

Radiative Equilibrium Expression for Atmospheric Temperature:

$$T_{\rm re}(\tau) = T_{\rm e}\left(\frac{1}{2} + \frac{\tau}{2\bar{\mu}}\right)^{1/4} \equiv T_{\rm e}\mathcal{G}^{1/4}(\tau) \tag{21}$$

where $\mathcal{G}(\tau) = (T_{\rm re}(\tau)/T_{\rm e})^4 = (1/2 + \tau/2\bar{\mu})$ is the **greenhouse factor**. Thus:

- the temperature increases monotonically downward from an outer 'skin' temperature $T_{\rm re}(0) = T_{\rm e}/(2)^{1/4}$ to a lower boundary temperature $T_{\rm re}(\tau^*) = T_{\rm re}(0)(1 + \frac{\tau^*}{\bar{\mu}})^{1/4}$ whose value is less than $T_{\rm s} = T_{\rm e}(1 + \tau^*/2\bar{\mu})^{1/4}$.

The relative temperature change (discontinuity) across the air-surface interface is:

$$\Delta T/T_{\rm s} = [T_{\rm s} - T_{\rm re}(\tau^*)]/T_{\rm s} = 1 - \left[\frac{1 + \tau^*/\bar{\mu}}{2 + \tau^*/\bar{\mu}}\right]^{1/4}. \tag{22}$$

The above expression for the surface temperature $T_{\rm s} = T_{\rm e}(1 + \tau^*/2\bar{\mu})^{1/4}$ can be found as follows.

The upward irradiance at the surface

$$F^+(\tau^*) = 2\pi\bar{\mu}I^+(\tau^*) = 2\bar{\mu}\sigma_{\mathrm{B}}T_{\mathrm{s}}^4 \quad \longleftarrow \quad I^+(\tau^*) = \sigma_{\mathrm{B}}T_{\mathrm{s}}^4/\pi.$$

Second, since the net irradiance at this level is $2\bar{\mu}\sigma_{\mathrm{B}}T_{\mathrm{e}}^4$ (as it is at all levels), then:

$$F(\tau^*) = F^+(\tau^*) - F^-(\tau^*) = 2\pi\bar{\mu}[I^+(\tau^*) - I^-(\tau^*)] = 2\bar{\mu}\sigma_{\mathrm{B}}T_{\mathrm{e}}^4.$$

Solving for the downward irradiance at the surface $[F^-(\tau^*) = F^+(\tau^*) - F(\tau^*)]$:

$$F^-(\tau^*) = 2\pi\bar{\mu}I^-(\tau^*) = 2\bar{\mu}\sigma_{\mathrm{B}(}T_{\mathrm{s}}^4 - T_{\mathrm{e}}^4) \quad \longrightarrow \quad I^-(\tau^*) = \sigma_{\mathrm{B}}(T_{\mathrm{s}}^4 - T_{\mathrm{e}}^4)/\pi. \tag{23}$$

Thus, the source function at the surface may be expressed as:

$$B(\tau^*) = \bar{I}(\tau^*) = \frac{1}{2}[I^+(\tau^*) + I^-(\tau^*)] = \frac{\sigma_{\mathrm{B}}}{2\pi}(2T_{\mathrm{s}}^4 - T_{\mathrm{e}}^4).$$

Evaluating Eq. 20 at $\tau = \tau^*$, we have

$$B(\tau^*) = \frac{\sigma_{\mathrm{B}}T_{\mathrm{e}}^4}{2\pi}(1 + \tau^*/\bar{\mu}).$$

Equating the two expressions above for $B(\tau^*)$, and solving for T_{s} we find:

$$\boxed{T_{\mathrm{s}} = T_{\mathrm{e}}(1 + \tau^*/2\bar{\mu})^{1/4} \equiv T_{\mathrm{e}}\mathcal{G}_{\mathrm{s}}^{1/4} \quad \longleftarrow \quad \textbf{surface temperature}.} \tag{24}$$

The value of the 'jump': $\Delta T/T_\mathrm{s} = 1 - [\frac{1+\tau^*/\bar{\mu}}{2+\tau^*/\bar{\mu}}]^{1/4}$ is $\sim$ 16% for optically-thin media, and decreases to zero as $\tau^* \to \infty$. This peculiarity arises from the fact that:

- the surface is heated by both the Sun and the atmosphere, whereas the overlying atmospheric layer is heated only by the neighboring regions.

Eq. (21) $[T_\mathrm{re}(\tau) = T_\mathrm{e}\left(\frac{1}{2} + \frac{\tau}{2\bar{\mu}}\right)^{1/4} \equiv T_\mathrm{e}\mathcal{G}^{1/4}(\tau)]$ ignores the possibility of dynamical heat transport across the interface. In the real world, **convection** tends to:

- erase the discontinuity extremely quickly, but not necessarily eliminate it.

The frequency-integrated IR optical depth at height z is given approximately by:

$$\tau(z) = \tau^* e^{-z/H_\mathrm{a}} = \tau^* e^{-z/2} \quad \Rightarrow \quad z = H_\mathrm{a} \ln(\tau^*/\tau) \tag{25}$$

where z is expressed in km above the surface ($z = 0$), H_a is the absorber scale height, and

- the total optical depth τ^* is an **adjustable parameter**, derived from **fitting model predictions to observed temperature profiles.**

Role of Radiation in Climate: Radiative Equilibrium (9)

The variation of temperature with height from Eq. 21: $T_{\rm re}(\tau) = T_{\rm e}\left(\frac{1}{2} + \frac{\tau}{2\bar{\mu}}\right)^{1/4}$ is shown in Fig. 13 (as the dashed lines) for several values of τ^*. Equation 24:

$T_{\rm s} = T_{\rm e}(1 + \dfrac{\tau^*}{2\bar{\mu}})^{1/4} \equiv T_{\rm e}\mathcal{G}_{\rm s}^{1/4}$ requires that: $\tau^*/2\bar{\mu} = 0.63$ to obtain the current globally averaged surface temperature of 288 K (assuming $T_{\rm e} = 255$ K).

This radiative equilibrium solution explains the declining tropospheric temperature:

- although a temperature inversion occurs in the upper stratosphere, due to absorption of solar UV radiation by ozone. But (see Fig. 13):

$$T_{\rm re}(0) = 255 \times 2^{-1/4}\ {\rm K} = 214\ {\rm K} \quad \longleftarrow \quad \text{the `skin' temperature}$$

is a good estimate for the globally-averaged minimum (tropopause) temperature.

- Note: $\tau^*/2\bar{\mu} = 0.63 \Rightarrow T_{\rm s} = 288$ K (15°C): this modest optical thickness causes
- a **greenhouse warming of 33°C**, relative to the effective temperature ($T_{\rm e} =$ 255 K or -18°C which would apply to an airless planet), **that prevents Earth from being a permanently glaciated world.**

Role of Radiation in Climate: Radiative Equilibrium (10)

In other words:

- **the existence of a tropopause temperature minimum can be understood from purely radiative considerations,** but the **lapse rate** in the troposphere is **controlled** largely **by dynamical transport** effects.

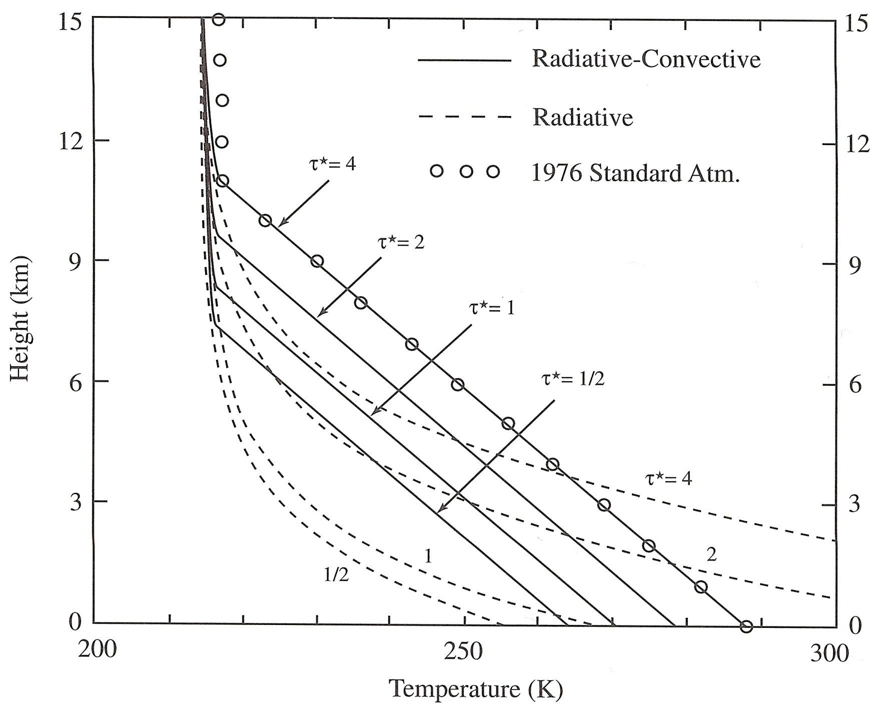


Figure 13: **Pure-radiative (dashed lines) and radiative-convective equilibrium (solid lines) temperature profiles for four different optical depths, and for $\bar{\rho} = 0.30$. Open circles: Standard Atmosphere.**

Radiation and Climate: Radiative-Convective Equilibrium (1)

In 1913, R. Emden first pointed out that: Radiative equilibrium (RE) solutions

- (such as those discussed above) are convectively unstable.

When solar energy is absorbed deep within the atmosphere, the RE temperature gradient or **lapse rate**, $\Gamma_{\rm re} = \partial T_{\rm re}/\partial z$ is large and negative. Instead we use:

The global mean tropospheric temperature gradient (or lapse rate, Fig. 13):

- $\Gamma_{\rm env} \approx -6.5 \;\; {\rm K} \cdot {\rm km}^{-1}$ $\longleftarrow$ the **environmental lapse rate**.

A proper treatment of convective transport is very difficult, but it is sufficient to:

- **replace RE lapse rate, $\Gamma_{\rm re}$, by environmental lapse rate, $\Gamma_{\rm env}$.**

To further simplify the problem, we introduce the **emission height** as:

- the level where the atmosphere experiences its maximum radiative cooling.
- The maximum cooling occurs at an optical depth $\tau_{\rm e}$, where $T \approx T_{\rm e}$.
- Its value in the RE problem discussed above is found from Eq. 21:

$$T_{\rm re}(\tau) = T_{\rm e}\left(\frac{1}{2} + \frac{\tau}{2\bar{\mu}}\right)^{1/4} \text{ by setting } T_{\rm re}(\tau_{\rm e}) = T_{\rm e}, \text{ which yields:}$$

- $\tau_{\rm e} = \bar{\mu}$: physically reasonable; a medium effectively cools when $\tau/\bar{\mu} \approx 1$.

- The geometric height is found from Eq. 25 to be $z_e(\tau^*) = H_a \ln(\tau^*/\tau_e)$.

We set the greenhouse increase in surface temperature T_s (over the effective temperature T_e) equal to the product of the lapse rate Γ_{env} and z_e, or:

$$T_s(\tau^*) = T_e + |\Gamma_{env}| z_e(\tau^*) = T_e + |\Gamma_{env}| H_a \ln(\tau^*/\tau_e). \tag{26}$$

We may now combine the radiative-convective solution valid in the region below the tropopause height, z_t and the RE solution valid above z_t:

$$T_{rc}(z) = T_e + |\Gamma_{env}|[z_e - z] \qquad (z \le z_t) \tag{27}$$
$$T_{rc}(z) = T_e (2)^{-1/4} \qquad (z > z_t) \tag{28}$$

where the tropopause height is given by $z_t = z_e + [1 - (\frac{1}{2})^{1/4}] \times T_e/|\Gamma_{env}|$.

- **We have obtained a realistic solution for the mean atmospheric temperature (see Fig. 13), in terms of the optical depth $\tau^* = 4$, lapse rate $|\Gamma_{env}| = 6.5 \ \ \mathrm{K \cdot km^{-1}}$, absorber scale height, $H_a = 2$ km, and $\tau_e = 0.32$, which yields emission height $z_e = 5$ km.**

Radiation and Climate: Radiative-Convective Equilibrium (3)

Using Eq. 26: $T_s(\tau^*) = T_e + |\Gamma_{env}| z_e(\tau^*) = T_e + |\Gamma_{env}| H_a \ln(\tau^*/\tau_e)$, "tune"

- τ^* to simulate the greenhouse effect in a realistic atmosphere. We may write:

τ^* as a sum of the opacities of the non-water greenhouse gases ($\tau_n^* = 0.788$) and:

- $\tau^*(H_2O) = bw$, due to water vapor, where b is an empirical constant and $w = \rho_0 H_a$ is the **precipitable water** [g$\cdot$ cm^{-2}]. Thus:

$$\tau_{tot}^* = \tau_n^* + bw = \tau_n^* + \tau^*(H_2O) \tag{29}$$

and Eq. 26: $T_s(\tau^*) = T_e + |\Gamma_{env}| z_e(\tau^*)$ now becomes:

$$\boxed{T_s(\tau_{tot}^*) = T_e + |\Gamma_{env}| z_e(\tau_{tot}^*) \qquad z_e(\tau_{tot}^*) = H_a \ln(\{\tau_n^* + \tau^*(H_2O)\}/\tau_e).} \tag{30}$$

To determine τ_n^* and b in our model, we impose two clear-sky constraints:

- (1) the greenhouse effect for $w = 0$: $G \equiv \sigma_B(T_s^4 - T_e^4) \approx 50$ W$\cdot$ m^{-2}; and
- (2) the greenhouse factor $\mathcal{G}_s = (T_s/T_e)^4 = [1 + |\Gamma_{env}| z_e(\tau_{tot}^*)/T_e]^4$ (Eq. 30) is consistent with observations.
- Constraint (1) comes from detailed modeling studies and constraint (2) from data from the Earth Radiation Budget Experiment (ERBE).

Radiation and Climate: Radiative-Convective Equilibrium (4)

A global average value of $\bar{w} = 1.32 \text{ g} \cdot \text{cm}^{-2}$, and $b = 1.1 \text{ cm}^2 \cdot \text{g}^{-1}$ implies:

$$\tau^*(\text{H}_2\text{O}) = b\bar{w} = 1.452 \quad \longrightarrow \quad \tau^*_{\text{tot}} = \tau^*_{\text{n}} + \tau^*(\text{H}_2\text{O}) = 0.788 + 1.452 = 2.21.$$

- Thus, the combination of non-water vapor greenhouse gases (CO_2, CH_4, N_2O, O_3 and others) contribute $\tau^*_{\text{n}}/\tau^*_{\text{tot}} = [0.788/2.21] = 0.35$ or $\sim$ 35% of the total clear-sky optical depth.

We find the greenhouse effect to be:

- $$G = \sigma_{\text{B}}[T_{\text{s}}^4(\tau^*_{\text{n}}) - T_{\text{e}}^4] = 53 \text{ W} \cdot \text{m}^{-2}$$

  by using $\tau^*_{\text{n}} = 0.788$, $\Gamma_{\text{env}} = -6.5 \text{ K} \cdot \text{km}^{-1}$ and $H_{\text{a}} = 2$ km in Eq. 26,

and the greenhouse factor (Eq. 30):

- $$\mathcal{G}_{\text{s}} = (T_{\text{s}}/T_{\text{e}})^4 = [1 + |\Gamma_{\text{env}}| z_{\text{e}}(\tau^*_{\text{tot}})/T_{\text{e}}]^4$$

  agrees reasonably well with ERBE results as shown in Fig. 14, provided the lapse rate is about $-6.5 \text{ K} \cdot \text{km}^{-1}$.

Radiation and Climate: Radiative-Convective Equilibrium (5)

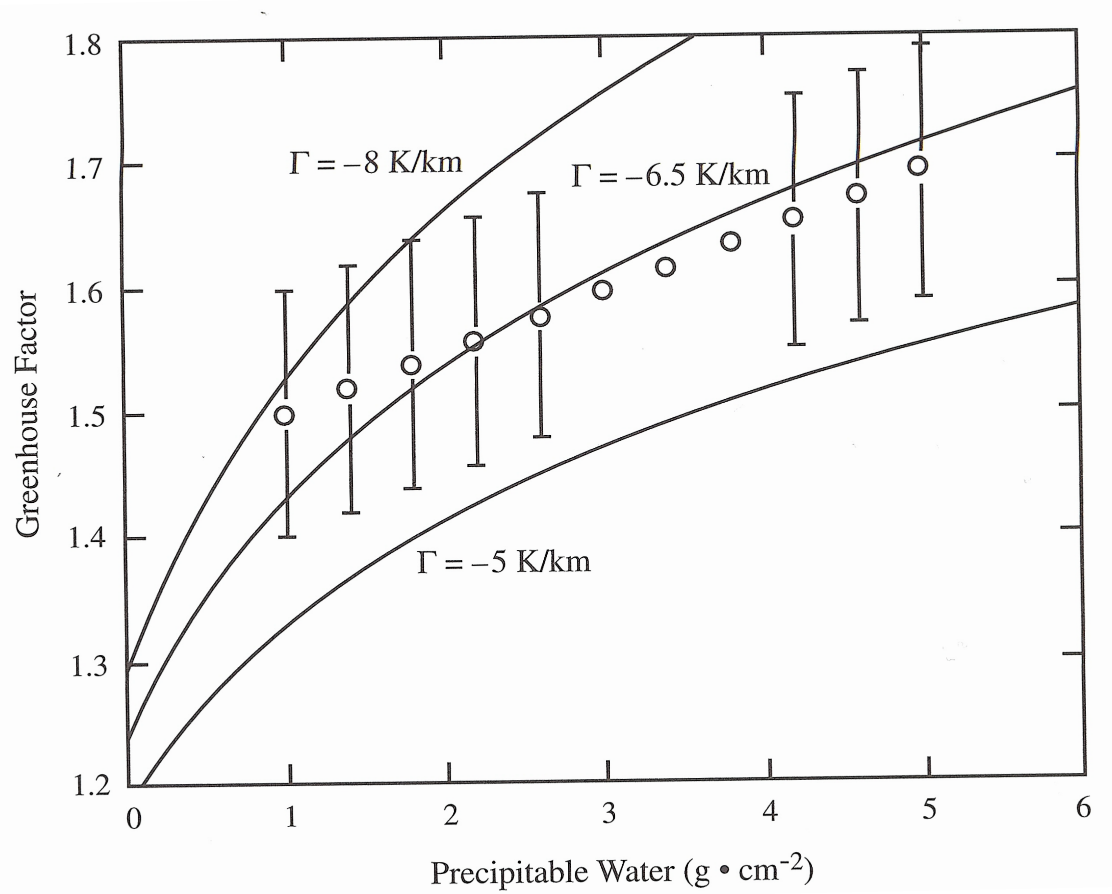


Figure 14: **Solid curves: Greenhouse factor $\mathcal{G}_{\rm s} = (T_{\rm s}/T_{\rm e})^4 = [1 + |\Gamma_{\rm env}| z_{\rm e}(\tau^*_{\rm tot})/T_{\rm e}]^4$ (Eq. 30) as a function of precipitable water for three different values of the environmental lapse rate $\Gamma \equiv \Gamma_{\rm env}$. Open circles: $\mathcal{G}$-values inferred from satellite data. The error bars indicate the spread of the data points.**

Role of Radiation in Climate: Impact of Clouds

For the combined shortwave and longwave radiative effects of clouds, the climatic effects of increasing greenhouse gas concentrations can be:

- **amplified**, if changes in clouds lead to a **warming** due to an **enhanced greenhouse effect**, or
- **dampened**, if changes in clouds lead to a **cooling** due to an **increased shortwave albedo**.

The absorption and scattering coefficients, and the asymmetry factor may be related to the LWC (IWC) and $\langle r \rangle$ by simple algebraic relationships:

- determined by fitting them to accurate scattering computations.

From these specifications, and by knowing the temperature of the cloud (determined by its height) we can compute:

- the cloud albedo, its emittance and absorptance, its optical depth, etc. In the following, we will:
- illustrate how these cloud properties enter into the mean radiative energy balance with the help of simple models.

Role of Radiation in Climate: IR Cloud Effects (1)

The IR opacity of clouds adds to that of the gaseous opacity:

- further 'blanketing' the surface and inducing additional warming.

We approximate the cloud opacity τ_c^* as the product of:

- the mass absorption coefficient $\alpha_{\rm IR}^{\rm c}$ and the liquid water path LWP:

$$\tau_c^* = \alpha_{\rm IR}^{\rm c}\,{\rm LWP} \qquad \longleftarrow \qquad \textbf{cloud IR optical depth.}$$

If the cloud is introduced **above** the clear-air emission height, it will:

- **raise the effective emitting level** to a cooler region, with a lower cooling rate, which implies a **higher surface temperature**. Therefore:
- we modify the effective cooling height $z_{\rm e}$ in Eq. 30: $T_{\rm s}(\tau_{\rm tot}^*) = T_{\rm e} + |\Gamma_{\rm env}| z_{\rm e}(\tau_{\rm tot}^*)$ to include the additional cloud optical depth:

$$\begin{aligned} T_{\rm s}(\tau_{\rm tot}^*) &= T_{\rm e} + |\Gamma_{\rm env}| z_{\rm e}(\tau_{tot}^*) \\ z_{\rm e}(\tau_{\rm tot}^*) &= H \ln[\{\tau_{\rm n}^* + \tau^*({\rm H_2O}) + \tau_{\rm c}^*\}/\tau_{\rm e}] \end{aligned} \tag{31}$$

Role of Radiation in Climate: IR Cloud Effects (2)

Figure 15 shows $\mathcal{G}_s = (T_s/T_e)^4$ derived from satellite data and predicted by Eq. 31 with $\tau_c^* = \alpha_{IR}^c$ LWP for two different values of α_{IR}^c. Note:

- **the principal IR effect of clouds** (introduced above the effective clear-air emission height) **is simply to raise the effective radiating height.**

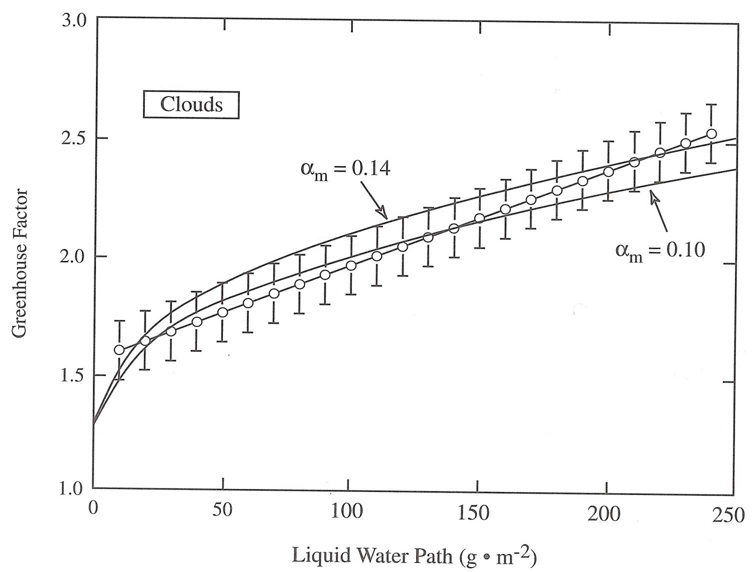


Figure 15: **Greenhouse factor $\mathcal{G}_s$ for a cloudy atmosphere versus LWP. Open circles: $\mathcal{G}_s$-values inferred from satellite data with error bars. The solid curves are for the analytic expression, Eq. 31, for two different values of the cloud droplet absorption coefficient, $\alpha_m \equiv \alpha_{IR}^c$.**

Role of Radiation in Climate: Combined IR Effects of Clouds and Clear Air

Combining the contributions from clouds (Eq. 31) and from clear air (Eq. 30), we get:

$$\begin{aligned} T_{\rm s} &= T_{\rm e} + A_{\rm c} H_{\rm cld} |\Gamma_{\rm cld}| \ln[(\tau_{\rm n}^* + \tau^*({\rm H_2O}) + \tau_{\rm c}^*)/\tau_{\rm e}] \\ &\quad + (1 - A_{\rm c}) H_{\rm clr} |\Gamma_{\rm clr}| \ln[(\tau_{\rm n}^* + \tau^*({\rm H_2O}))/\tau_{\rm e}]. \end{aligned} \tag{32}$$

Note that:

- we have taken into account the fact that the Earth is partially covered with clouds with a fractional coverage $A_{\rm c}$ $(0 \leq A_{\rm c} \leq 1)$;
- both the scale height (either $H_{\rm clr}$ or $H_{\rm cld}$) and the lapse rate (either $\Gamma_{\rm clr}$ or $\Gamma_{\rm cld}$) are allowed to vary between clear and cloudy regions;
- the above equation may be used to study how the surface temperature of a planet depends on variable cloud cover, $A_{\rm c}$, moisture content of the air, $\tau^*({\rm H_2O})$, optical depth due to non-water vapor GHGs, $\tau_{\rm n}^*$, and cloud water optical depth, $\tau_{\rm c}^*$.

Role of Radiation in Climate: SW Cloud Effects (1)

The major impact on the energy budget is the change in **albedo** due to clouds:

- increased reflection (larger $\bar{\rho}$) causes the amount of absorbed energy $(1-\bar{\rho})F^{\mathrm{s}}$ to be lowered, and thus reduces the mean surface temperature.

The plane albedo ρ_{c} of a stratified cloud can be computed from the **TSA** as:

$$\rho_{\mathrm{c}} = \frac{2b_{\mathrm{c}}\tau_{\mathrm{c}}^{*} + (\bar{\mu}-\mu_0)(1-e^{-2b_{\mathrm{c}}\tau_{\mathrm{c}}^{*}/\mu_0})}{2b_{\mathrm{c}}\tau_{\mathrm{c}}^{*} + 2\bar{\mu}} \quad \longleftarrow \quad \textbf{plane cloud albedo.} \tag{33}$$

Here

- $\tau_{\mathrm{c}}^{*} = \alpha_{\mathrm{VIS}}^{\mathrm{c}}\,\mathrm{LWP}$ is the (gray) visible optical depth of the cloud,
- $\bar{\mu}$ is the mean cosine of the multiply-scattered light,
- μ_0 is the cosine of the solar zenith angle,
- $b_{\mathrm{c}} = (1-g_{\mathrm{c}})/2$ = cloud backscattering ratio, and g_{c} = asymmetry factor.
- Figure 16 shows: cloud albedo-values extracted from Nimbus 7 and ERBE satellite data, and for comparison predictions from Eq. 33.

Role of Radiation in Climate: SW Cloud Effects (2)

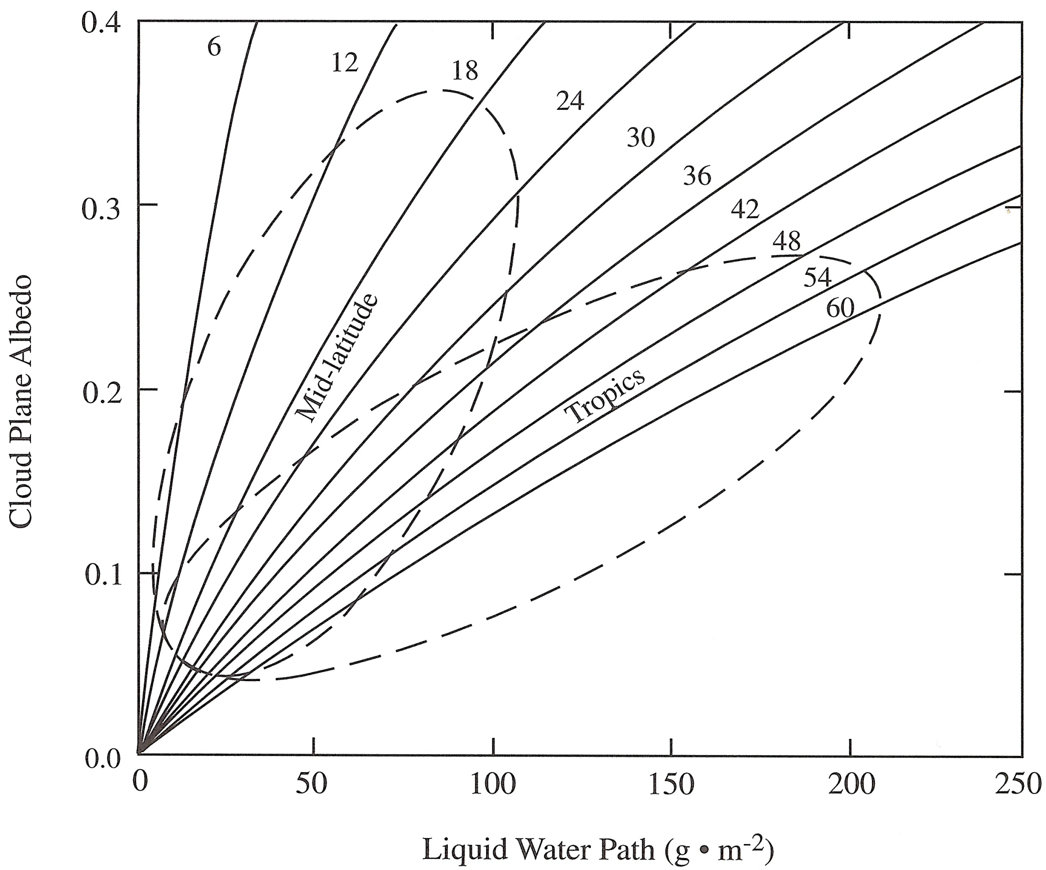


Figure 16: **Plane albedo of a cloud versus liquid water path, calculated from Eq. 33. Different curves apply to different values of the mean particle radius $\langle r \rangle$ in μm. The dashed curves, based on satellite data, enclose 97% of all observations for the tropics and mid-latitudes.**

Role of Radiation in Climate: SW Cloud Effects (3)

How does the albedo of the **surface-atmosphere-cloud "system"** affect the overall energy balance of the planet? First we should recognize that:

- the Earth's surface reflects visible light, with an average (spherical) albedo $\bar{\rho}_{\rm s} = 0.11$. This value is low, due to:
- the very low (5-10%) reflectance of ocean water which covers nearly three-fourth of the planet's surface. We must also include:
- the small, but still significant effects of the clear-sky (molecular) scattering and absorption of solar radiation;
- the spherical albedo of the atmosphere (assuming a dark surface) $\bar{\rho}_{\rm a} = 0.07$.

Spherical albedo of the combined **clear-sky** atmosphere-surface system:

$$\boxed{\bar{\rho}_{\rm as} = \bar{\rho}_{\rm a} + \frac{\bar{\rho}_{\rm s}(1-\bar{\rho}_{\rm a})^2}{1-\bar{\rho}_{\rm s}\bar{\rho}_{\rm a}} \quad \longleftarrow \quad \textbf{clear-sky albedo}. \qquad (34)}$$

Substituting in Eq. 34 numerical values for $\bar{\rho}_{\rm a} = 0.07$ and $\bar{\rho}_{\rm s} = 0.11$:

- we find $\bar{\rho}_{\rm as} = 0.166$, in good agreement with the observed range $0.14 - 0.18$ found by satellite measurements.

Role of Radiation in Climate: SW Cloud Effects (4)

Assuming that clouds of spherical albedo $\bar{\rho}_{\rm c}$ are present, we can apply the same formula to estimate the **spherical albedo** of the **cloudy** atmosphere-surface system:

$$\bar{\rho}_{\rm cas} = \bar{\rho}_{\rm c} + \frac{\bar{\rho}_{\rm as}(1-\bar{\rho}_{\rm c})^2}{1-\bar{\rho}_{\rm as}\bar{\rho}_{\rm c}} \quad \longleftarrow \quad \textbf{cloudy-sky albedo}. \tag{35}$$

To include partial cloudiness with a fractional coverage $A_{\rm c}$ $(0 \leq A_{\rm c} \leq 1)$:

- we weight the clear-sky albedo $\bar{\rho}_{\rm as}$ with the fraction of clear sky $(1 - A_{\rm c})$ so that the total spherical albedo of the Earth system is approximated by:

$$\bar{\rho}_{\rm tot} = A_{\rm c}\bar{\rho}_{\rm cas} + (1 - A_{\rm c})\bar{\rho}_{\rm as}. \tag{36}$$

We would like to relate the cloud spherical albedo $\bar{\rho}_{\rm c}$ to more basic:

- cloud properties, such as liquid water path (LWP) or ice water path (IWP).

In the **TSA**, $\bar{\rho}_{\rm c}$ is given by averaging Eq. 33: $\rho_{\rm c} = \frac{2b_{\rm c}\tau_{\rm c}^* + (\bar{\mu}-\mu_0)(1-e^{-2b_{\rm c}\tau_{\rm c}^*/\mu_0})}{2b_{\rm c}\tau_{\rm c}^* + 2\bar{\mu}}$ over all solar zenith angles, BUT, simply setting $\mu_0 = \bar{\mu}$ in Eq. 33 yields the correct result:

$$\bar{\rho}_{\rm c} = \frac{2b_{\rm c}\tau_{\rm c}^*}{2b_{\rm c}\tau_{\rm c}^* + 2\bar{\mu}} \quad \longleftarrow \quad \textbf{spherical cloud albedo}. \tag{37}$$

Role of Radiation in Climate: Combined SW and IR Effects of Clouds (1)

If we define a critical albedo (referenced to a clear atmosphere with albedo $\bar{\rho}_{\rm as}$):

$$\bar{\rho}_{\rm crit} = \frac{\bar{\rho}_{\rm as} + a\,{\rm LWP}}{1 + a\,{\rm LWP}}; \qquad a = 2.622 \times 10^{-3} \tag{38}$$

then clouds will result in:

- a net **cooling** if $\bar{\rho}_{\rm tot} > \bar{\rho}_{\rm crit}$ BUT
- a net **warming** will occur, if $\bar{\rho}_{\rm tot} < \bar{\rho}_{\rm crit}$.

This result is illustrated in Fig. 17 which shows that: the critical albedo separates domains of cooling and warming:

- clouds in mid-latitude regions tend to have a net cooling effect, whereas those in tropical regions are mainly in the warming domain.

To obtain an alternate model for the surface temperature we use the concept of:

- **effective** radiating height in a formulation involving partial cloud coverage.

Both the albedo and the **effective radiating height** depend upon:

- the LWP of the clouds, and upon the cloud fractional coverage, $A_{\rm c}$.

Role of Radiation in Climate: Combined SW and IR Effects of Clouds (2)

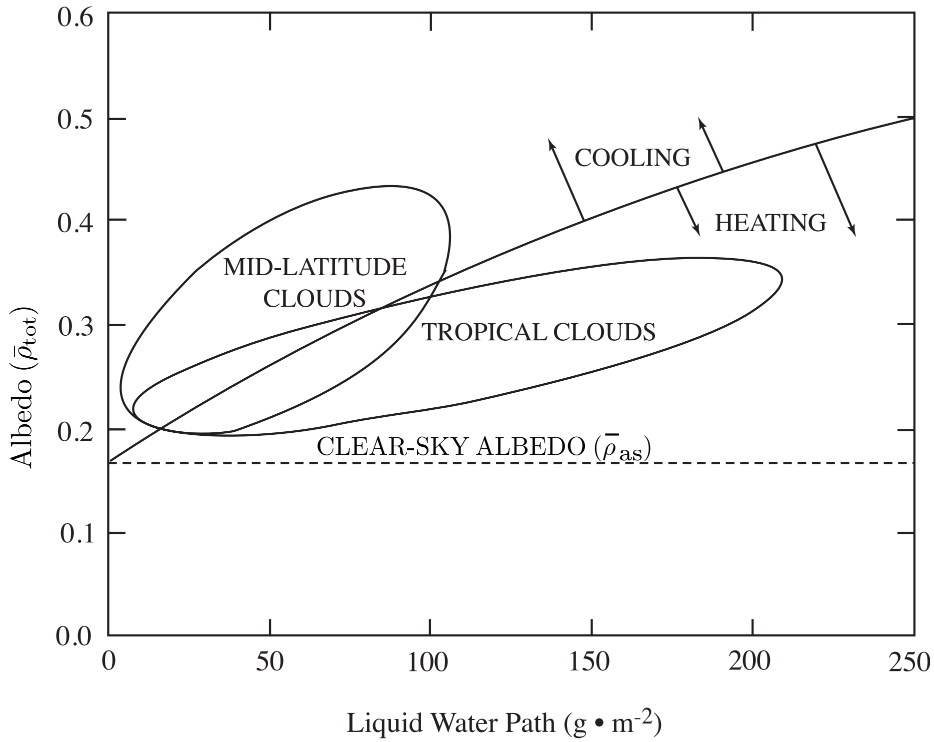


Figure 17: **Albedo versus liquid water path of clouds. Heavy solid line: Critical planetary albedo $\bar{\rho}_{crit}$ dividing regions of net warming and net cooling, given by Eq. 38. The albedo of the clear-air plus surface $\bar{\rho}_{as}$ is set equal to 0.166, as inferred from Eq. 34. The ellipses define the regions containing 97% of all observations for the tropics and midlatitudes. They were computed from Eq. 35 where $\bar{\rho}_{c}$ is given by the ellipses in Fig. 16. If $\bar{\rho}_{tot}$ is in the upper domain ($\bar{\rho}_{tot} > \bar{\rho}_{crit}$), net cooling occurs. If it is in the lower domain, ($\bar{\rho}_{tot} < \bar{\rho}_{crit}$), net warming occurs.**

Role of Radiation in Climate: Radiative Forcing (1)

We have so far defined the greenhouse effect G to be:

- the difference between the planetary mean irradiance emitted by the surface and the planetary mean outgoing irradiance at the top of the atmosphere.

But we may also define it on a local basis, and at a specific wavenumber as:

$$G_{\tilde{\nu}} \equiv F^{+}_{\tilde{\nu}}(\tau^{*}) - F^{+}_{\tilde{\nu}}(0).$$

The spectrally-integrated greenhouse effect is thus:

$$G = \int d\tilde{\nu} G_{\tilde{\nu}} = \sigma_{\mathrm{B}} T_{\mathrm{s}}^{4} - F_{\mathrm{TOA}}. \tag{39}$$

On a global average, $\bar{F}_{\mathrm{TOA}} = \sigma_{\mathrm{B}} T_{\mathrm{e}}^{4}$. Figure 18 shows:

- the spectral variation of these two irradiances, illustrating how their difference maximizes in the optically thick bands of the major greenhouse gases.
- A change in a climate variable will cause a perturbation ΔG in G, which gives rise to a change ΔT_{s} in the equilibrium surface temperature T_{s}.

Role of Radiation in Climate: Radiative Forcing (2)

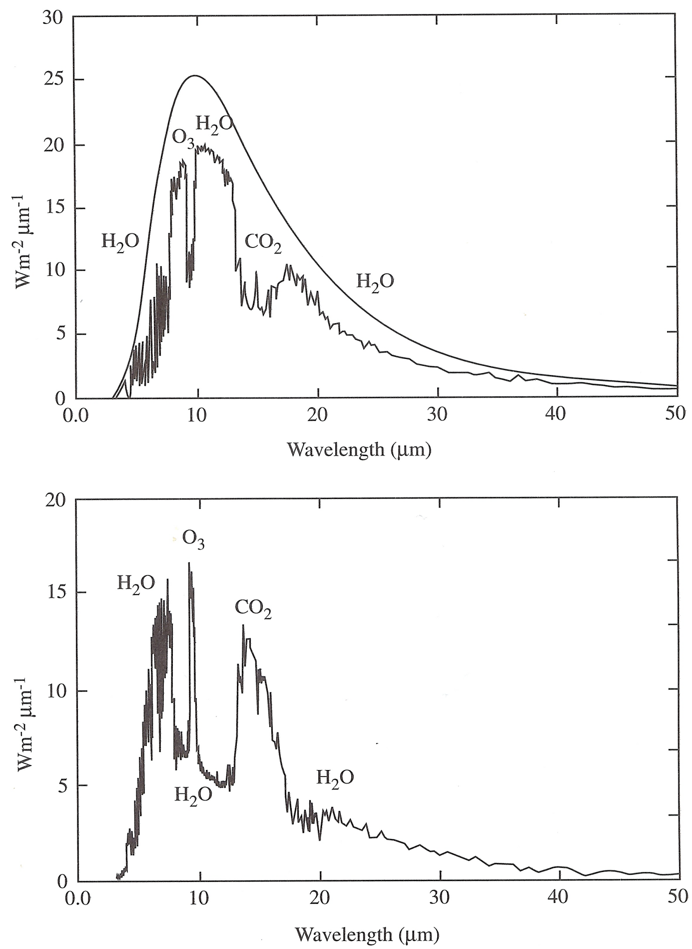


Figure 18: **Upper panel: The upper smooth curve is the emission from the surface (assumed black) The lower curve is the irradiance at the top of the atmosphere, computed from a radiative transfer model. Lower panel: The corresponding Greenhouse effect $G_\lambda = \sigma_B T_s^4 - F_{TOA}$.**

Role of Radiation in Climate: Radiative Forcing (3)

Drawbacks to the definition of ΔG, which limit its usefulness:

- (1) Since it involves surface temperature, G and ΔG include the feedback **response** of the climate system;
- (2) ΔG is a measure of change in atmospheric "back-warming" of the surface – a more satisfactory definition would include the entire atmospheric column;
- (3) T_s is not easily measured from space, especially over land surfaces.

Would prefer a strictly radiative definition such as **the difference between the incoming and outgoing irradiances at the top of the atmosphere**:

$$N \equiv N_{sw} - N_{lw} \equiv (1 - \rho)F^s - F_{TOA}. \tag{40}$$

N is equal to the instantaneous column-integrated radiative heating resulting from an imbalance between the shortwave heating and the longwave cooling. Note:

- this definition involves local (or regional quantities): N can vary with latitude and even time of day;
- the plane albedo ρ, the solar irradiance F^s, and the outgoing IR irradiance F_{TOA} are readily measured from space.

Role of Radiation in Climate: Radiative Forcing (4)

To illustrate the role of N as a radiative forcing, we consider the more general energy balance equation applying to a particular region:

$$\frac{\partial E_{\text{atm}}}{\partial t} = N - \int_0^\infty dz \, \text{div} F_{\text{h}} \equiv N - \Phi. \tag{41}$$

E_{atm} is the zonally- and column-averaged atmospheric energy. Φ is the zonally- and column-averaged irradiance divergence of energy leaving the atmospheric column horizontally.

If we average the above equation over the Earth's surface, the transport terms will drop out, so that $\partial \bar{E}_{\text{atm}}/\partial t = \bar{N}$. Further averaging over one year (or a period of several years) yields a global balance:

$$\frac{\partial \langle \bar{E}_{\text{atm}} \rangle}{\partial t} = \langle \bar{N} \rangle = 0. \tag{42}$$

- The requirement that $\langle \bar{N} \rangle = 0$ is a useful check on the accuracy of radiation budget measurements.
- Figure 19 shows the measured variation of $\langle N \rangle$ with latitude and the inferred meridional transport of energy $\langle \int dz F_{\text{h}} \rangle$, assuming $\langle \bar{N} \rangle = 0$. More detailed geographical distributions of N are found in the research literature.

Role of Radiation in Climate: Radiative Forcing (5)

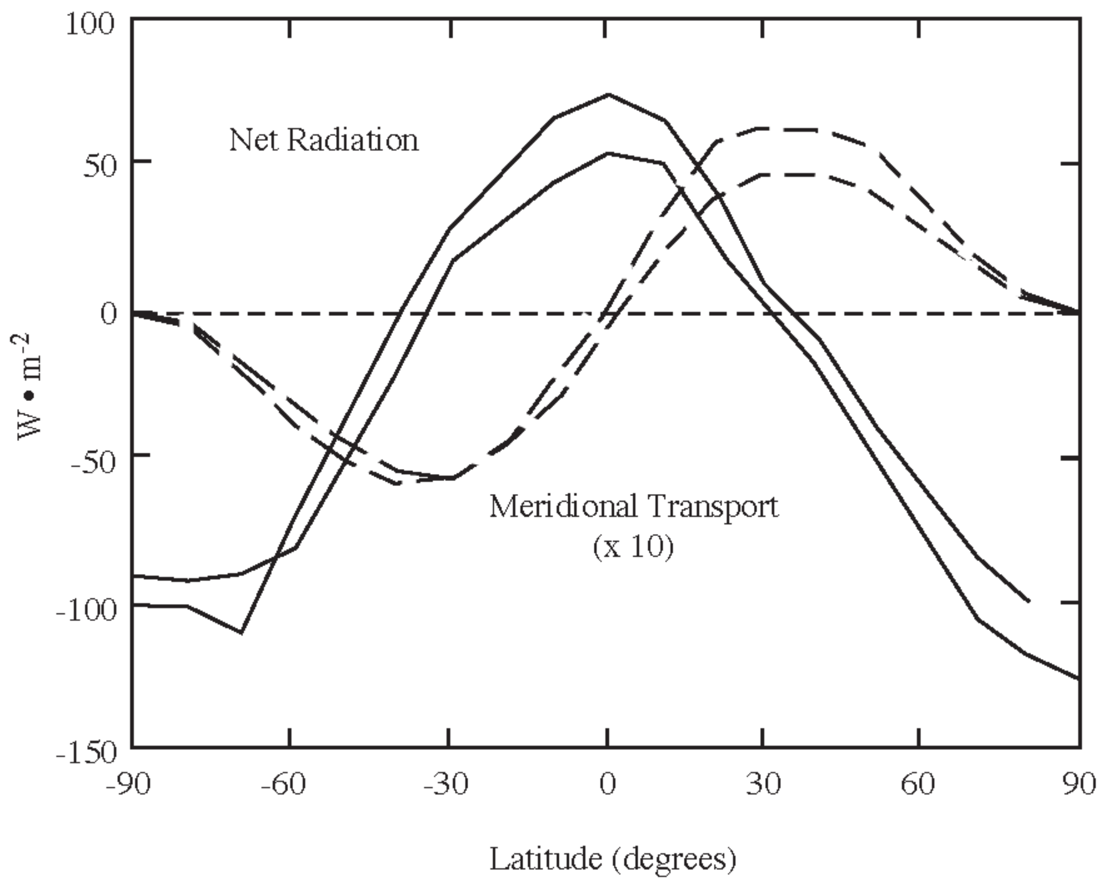


Figure 19: **Curves labelled "Net Radiation": Zonally and annually-averaged net irradiance $\langle N \rangle$ from satellite observations. Curves labelled "Meridional Transport" is the north-south transport $\int dz F_{\rm h}$ implied by the above variation of $\langle N \rangle$ with latitude, and assuming a balance between meridional transport and radiative heating, $\partial \langle \bar{E}_{\rm atm} \rangle / \partial t = 0$ (see Eq. 42).**

Role of Radiation in Climate: Cloud/Aerosol Forcing (1)

To focus on the radiative effects of clouds (same formulation applies to aerosols):

- it is common to use the concept of **cloud radiative forcing:**

$$\mathrm{CF} \equiv N_{\mathrm{cld}} - N_{\mathrm{clr}} \equiv \mathrm{SWCF} + \mathrm{LWCF}. \tag{43}$$

N_{cld} and N_{clr} are the cloudy and clear irradiances measured at the top of the atmosphere.

- If CF is determined empirically: use is made of different "scenes" (cloudy or clear) at the same latitude and above the same type of surface.‡

Cloud forcing CF is conveniently separated into shortwave (SWCF) and longwave (LWCF) contributions. Clearly:

$$\begin{aligned} \mathrm{SWCF} &\equiv (1-\rho_{\mathrm{cas}})F^{\mathrm{s}} - (1-\rho_{\mathrm{as}})F^{\mathrm{s}} = (\rho_{\mathrm{as}} - \rho_{\mathrm{cas}})F^{\mathrm{s}} \\ \mathrm{LWCF} &\equiv F_{\mathrm{TOA}}(\mathrm{clr}) - F_{\mathrm{TOA}}(\mathrm{cld}). \end{aligned} \tag{44}$$

- ρ_{cas} = plane albedo of the cloudy atmosphere (clouds + atmosphere + surface)
- ρ_{as} = plane albedo of the clear atmosphere (atmosphere + surface). Note that:
- since clouds usually increase the overall albedo, SWCF is generally negative;
- since clouds tend to reduce the outgoing irradiance, LWCF is generally positive.

‡By referencing the clear atmosphere: we isolate the effects of clouds from other radiative effects, for example, changes of water vapor or surface albedo.

In contrast with the greenhouse effect (Fig. 18), where the optically-thick bands dominate, Fig. 20 shows that the transparent window at 8–12 μm region dominates the IR cloud forcing.

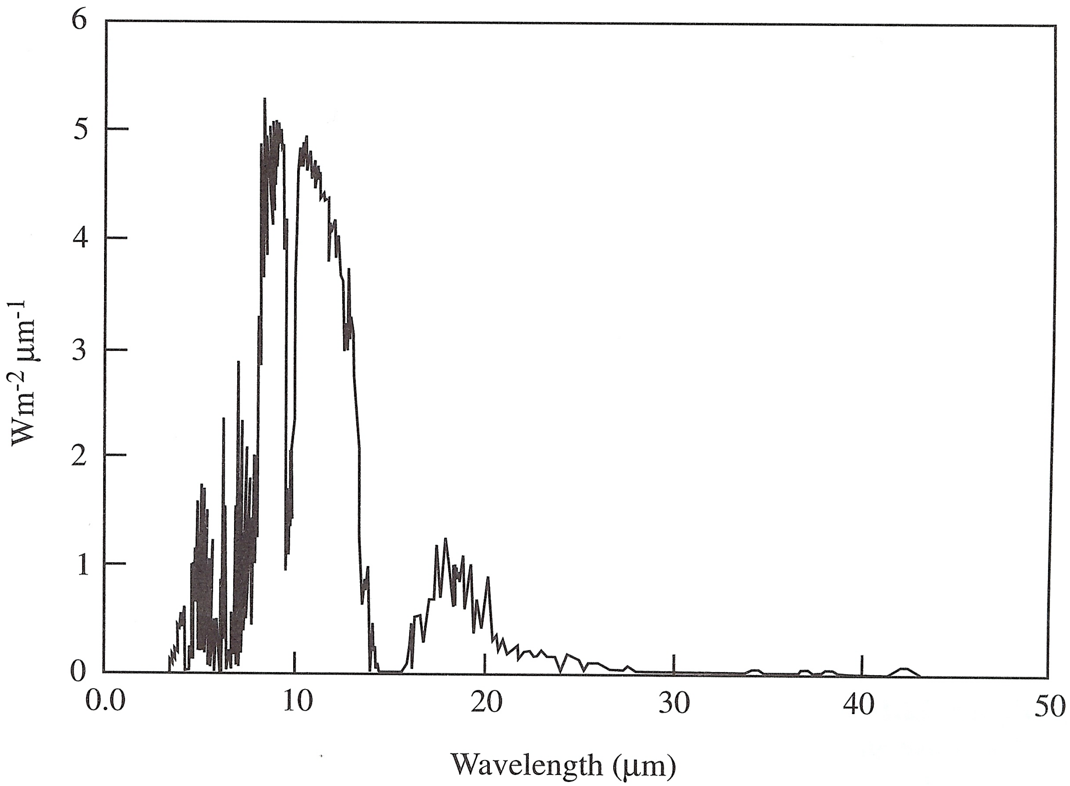


Figure 20: **Spectral variation of IR cloud forcing ($\mathrm{W{\cdot}m^{-2}\cdot\mu m^{-1}}$), the difference between the clear-air irradiance at the top of the atmosphere and the cloudy outgoing IR irradiance.**

The Role of Radiation in Climate: Aerosol Forcing (1)

- Important for climate are: particles which originate in the stratosphere as a result of an injection of sulfates (mainly SO_2) from volcanic eruptions.
- Sulfuric acid particles result from: heterogeneous nucleation on existing aerosols (Junge layer particles) and homogeneous nucleation in super-cooled regions.
- These liquid particles persist in the stratosphere for several years following major eruptions, such as El Chichon in 1982, and Mt. Pinatubo in 1991.

The optical properties are similar to those of small cloud particles in that they tend:

- to cool the atmosphere in the SW, and warm it in the IR spectral range, BUT
- if $r_{\text{eff}} > 2\ \mu$m, the tendency is to produce warming (see Fig. 21).

Since stratospheric aerosols of volcanic origin are in the size range 0.5–1.5 μm:

- the overall effect is to cool the Earth.

In fact, the largest eruption of this century, Mt. Pinatubo:

- caused a peak radiative forcing of 4–5 W$\cdot$ m^{-2}, which caused:
- a cooling in the northern hemisphere lower troposphere of 0.5–0.7°C, compared with pre-Pinatubo levels.

The Role of Radiation in Climate: Aerosol Forcing (2)

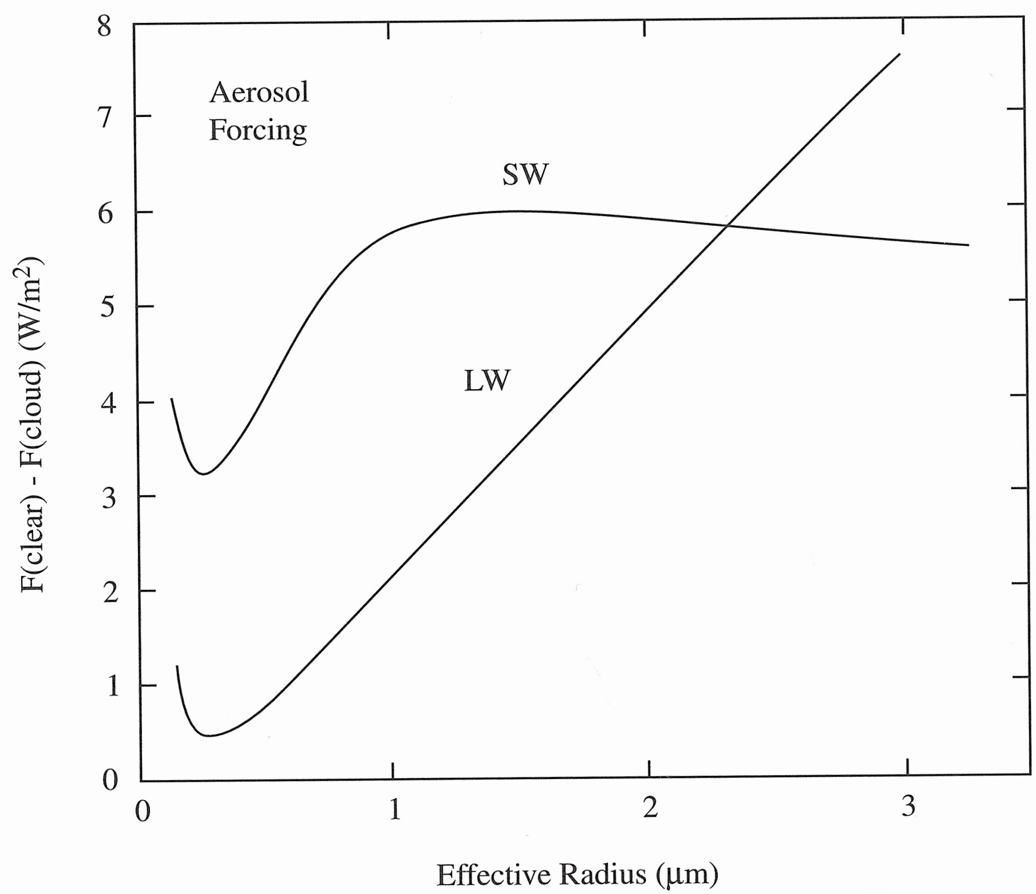


Figure 21: **Change of shortwave and longwave irradiances at the tropopause (longwave and shortwave forcing) caused by adding a stratospheric $(20 - 25$ km) aerosol layer with $\tau(0.55\ \mu\text{m}) = 0.1$ in a 1-D radiative-convective model with fixed surface temperature.**

The Role of Radiation in Climate: Water-vapor feedback (1)

We have already discussed [see Eq. (40)] the concept of:

- **radiative forcing:** $N = N_{\rm sw} - N_{\rm lw}$ as a quantitative measure of greenhouse warming; sw = shortwave, lw = longwave. In our simple model:
- $N_{\rm sw} = (1-\rho)F^{\rm s}$ and $N_{\rm lw} = F_{\rm TOA}$.

It can be shown that in a "zero-D" model, the $T_{\rm s}$ response to a forcing is written as the product of

- (i) a direct response $\Delta T_{\rm s}^{\rm d} = \frac{\Delta N}{(4F_{\rm TOA}/T_{\rm s})}$, where ΔN is the net TOA radiative forcing, which is zero in radiative equilibrium, and
- (ii) the gain $f = (1 - \Sigma_i\,\lambda_i)^{-1}$ of the system. Thus:

$$\Delta T_{\rm s} = f\,\Delta T_{\rm s}^{\rm d} = \frac{1}{(1-\Sigma_i\,\lambda_i)}\frac{\Delta N}{(4F_{\rm TOA}/T_{\rm s})}. \tag{45}$$

The various feedback parameters, λ_i, are measures of individual feedback processes, written in terms of a general temperature-dependent climate variable Q_i as:

$$\lambda_i = [T_{\rm s}/4F_{\rm TOA}]\frac{\partial N}{\partial Q_i}\frac{\partial Q_i}{\partial T_{\rm s}}. \tag{46}$$

The Role of Radiation in Climate: Water-vapor feedback (2)

The most important terrestrial feedback due to the change in IR opacity results from the temperature dependence of the water vapor content:

- the atmosphere becomes more (less) moist in a warmer (colder) climate resulting in more (less) greenhouse warming.

The temperature-moisture content relationship is described by:

- the **Clausius-Clapeyron equation**, expressing the dependence of the saturation vapor pressure on temperature.

An empirical relationship between the precipitable water w and the sea-surface temperature T_s has been determined from an analysis of satellite measurements:

$$w = a_w e^{b_w (T_\mathrm{s} - 288)} \qquad T_\mathrm{s}\ [\mathrm{K}],\ a_w = 1.753\ \mathrm{g \cdot cm^{-2}},\ b_w = 0.0686\ \mathrm{K^{-1}}. \tag{47}$$

This result is consistent with:

- an average surface relative humidity (RH) of 85% over the World's oceans, provided the mean scale height for water vapor is 2 km. Note that:

RH tends to remain fixed as the sea-surface temperature changes, whereas:

- the absolute water content w depends exponentially on temperature.

The Role of Radiation in Climate: Water-vapor feedback (3)

We now have the equations required to estimate water-vapor feedback in the clear-sky atmosphere. Recall the relationship (see Eq. 29) between τ^* and w:

$$\tau^* = \tau_{\rm n}^* + bw \qquad \tau_{\rm n}^* = 0.788$$

where $\tau_{\rm n}^*$ denotes the clear-sky optical depth due to non-water vapor GHGs, and b is a constant determined from fitting the model greenhouse factor to satellite data.

- Note: the temperature-dependent variable in Eq. 46 is $Q = \tau^*({\rm H_2O}) = bw$.

Since $N_{\rm lw} = \sigma_{\rm B} T_{\rm e}^4$, which can be expressed in terms of optical depth τ^* from Eq. 26, $N_{\rm lw} = \sigma_{\rm B}[T_{\rm s} - |\Gamma_{\rm env}| H_{\rm a} \ln(\tau^*/\tau_{\rm e})]^4$, we find:

$$\lambda = \left(\frac{T_{\rm s}}{4F_{\rm TOA}}\right)\left(\frac{\partial N}{\partial \tau^*}\right)\left(\frac{\partial \tau^*}{\partial w}\right)\left(\frac{\partial w}{\partial T_{\rm s}}\right) = \frac{|\Gamma_{\rm env}| H_{\rm a}(b)(b_w w)}{\tau_{\rm n}^* + bw}. \qquad (48)$$

- Note that $\lambda > 0$, indicating a **positive** feedback.

For $w = \bar{w} = 1.32$ g $\cdot$ cm^{-2}, and the values for $b = 1.1$ cm$^2 \cdot$ g^{-1}, $b_w = 0.0686$ K^{-1} etc., we find:

- $\lambda = 0.66$, in good agreement with numerical climate models, which yield values of λ in the range 0.59 to 0.77.

An approximate result for the CO_2 radiative forcing is (from IPCC):

$$N_{\rm lw}(\chi) = 32 + 6.3\ln(\chi/\chi_0) \qquad [{\rm W\cdot m^{-2}}] \tag{49}$$

χ_0 = present-day CO_2 concentration. For a CO_2-doubling, Eq. 49 predicts:

- $\Delta N_{\rm lw} = N_{\rm lw}(2\chi_0) - N_{\rm lw}(\chi_0) = 6.3\ln 2 = 4.37\ {\rm W\cdot m^{-2}}$.

The climate response predicted from Eq. 45 is, $\Delta T_{\rm s} = f\Delta T_{\rm s}^{\rm d} = 1.2f$ °C, and empirical data suggest that the gain falls in the range $2 < f < 4$. Thus:

- the rise in mean surface temperature for a CO_2-doubling is expected to be in the range 2.4–4.8 K, in agreement with accurate model predictions (1.5–4.5 K).

According to Eq. 45, $\Delta T_{\rm s}^{\rm d} = 0.36$ K over the $1900 - 1990$ period due to the known greenhouse forcing (Fig. 27) that has occurred over this period ($1.92\ {\rm W\cdot m^{-2}}$).

- With a gain $f = 1/(1-\lambda) = 1/(1-0.66) \approx 2.9$ from water vapor feedback, the predicted change of $\Delta T_{\rm s} = f\Delta T_{\rm s}^{\rm d} = 2.9 \times 0.36\ {\rm K} \approx 1$ K, exceeds the observed value (0.5 K).

A number of factors may be responsible for this discrepancy, BUT:

- At least part of the problem stems from the neglect of a **time lag** in the system response, due to the slow overturning of the mixed (upper) layer of the ocean.

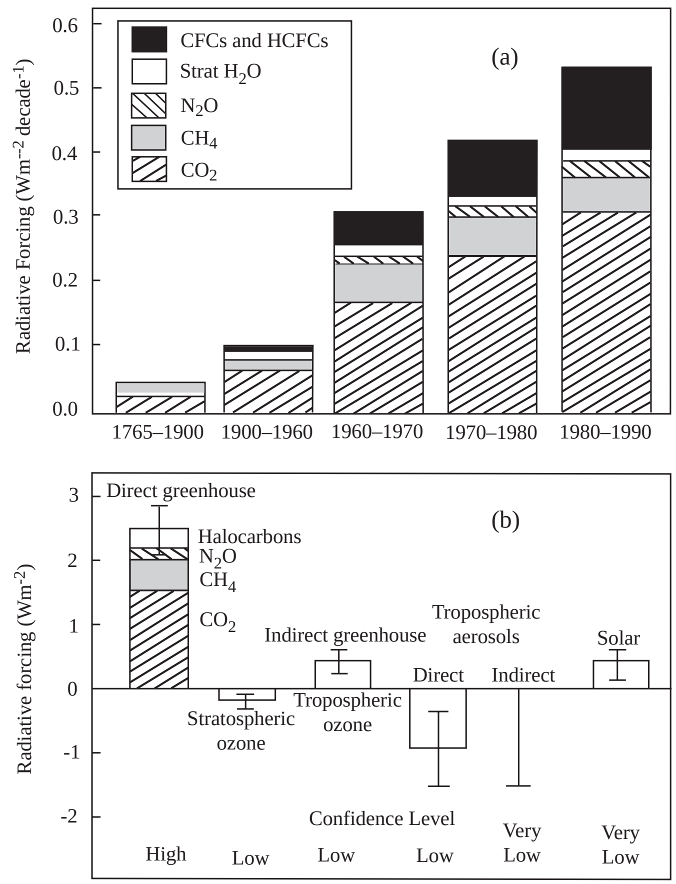


Figure 22: **(a) The degree of radiative forcing produced by selected greenhouse gases in five different epochs. Until about 1960, nearly all the forcing was due to CO_2; today the other greenhouse gases combined nearly equal the CO_2 forcing. (b) Estimates of the globally averaged radiative forcing due to changes in greenhouse gases and aerosols from pre-industrial times to the present day and changes in solar variability from 1850 to the present day.**

Role of Radiation in Climate: Radiative Equilibrium with Finite Visible Optical Depth (1)

Any realistic atmosphere absorbs radiation at both IR and visible wavelengths. How will shortwave absorption (ignored above) affect the mean radiative balance? We assume that the solar energy absorbed by a rotating planet is governed by:

- a diurnal average over all incident solar angles, so that the average cosine is $\bar{\mu}_0$;
- the average solar irradiance absorbed over a diurnal period is defined to be $\bar{\mu}_0 F_{\mathrm{a}}^{\mathrm{s}}$;§
- other than allowing for a finite spherical albedo, we ignore scattering as before.

The total (visible plus IR) net irradiance is written:

$$F_{\mathrm{tot}}(z) = -F_{\mathrm{V}}(z) + F_{\mathrm{IR}}(z) = -\bar{\mu}_0 F_{\mathrm{a}}^{\mathrm{s}} e^{-\tau_{\mathrm{V}}/\bar{\mu}_0} + F_{\mathrm{IR}}(z) \tag{50}$$

where τ_{V} is the frequency-averaged visible optical depth. Radiative equilibrium requires: $F_{\mathrm{tot}} =$ constant, or $dF_{\mathrm{tot}}/dz = 0$. Using the chain rule of differentiation:

$$\frac{dF_{\mathrm{tot}}}{dz} = -\frac{dF_{\mathrm{V}}}{dz} + \frac{dF_{\mathrm{IR}}}{dz} = -\frac{d(\bar{\mu}_0 F_{\mathrm{a}}^{\mathrm{s}} e^{-\tau_{\mathrm{V}}/\bar{\mu}_0})}{d\tau_{\mathrm{V}}}\frac{d\tau_{\mathrm{V}}}{dz} + \frac{dF_{\mathrm{IR}}}{dz}$$

$$= -k_{\mathrm{V}} F_{\mathrm{a}}^{\mathrm{s}} e^{-\tau_{\mathrm{V}}/\bar{\mu}_0} + \frac{dF_{\mathrm{IR}}}{dz} = 0$$

where k_{V} is the (gray) visible absorption coefficient, so that $d\tau_{\mathrm{V}} = -k_{\mathrm{V}} dz$.

§ $\bar{\mu}_0 F_{\mathrm{a}}^{\mathrm{s}} = (1 - \bar{\rho}) S_0$ is the absorbed solar irradiance.

Role of Radiation in Climate: Radiative Equilibrium with Finite Visible Optical Depth (2)

This equation provides us with the desired relationship between the infrared irradiance derivative and the solar heating rate:

$$\mathcal{H}_{\mathrm{V}} = -dF_{\mathrm{V}}/dz = k_{\mathrm{V}} F_{\mathrm{a}}^{\mathrm{s}} e^{-\tau_{\mathrm{V}}/\bar{\mu}_0}.$$

An expression relating the derivative of the IR irradiance to the source function is obtained from the divergence of the IR irradiance (see Eq. 5.75 in STS): applied to a gray, perfectly absorbing medium: $\mathcal{H}_{\mathrm{IR}} = -\partial F_{\mathrm{IR}}/\partial z = 4\pi k_{\mathrm{IR}}[\bar{I}(z) - B(z)]$:

$$B(\tau) = \bar{I} - \frac{1}{4\pi}\frac{\partial F_{\mathrm{IR}}}{\partial \tau} = \bar{I}(\tau) + \frac{F_{\mathrm{a}}^{\mathrm{s}}}{4\pi n} e^{-\tau/n\bar{\mu}_0}$$

where $d\tau \equiv -k_{\mathrm{IR}} dz$, and $n = k_{\mathrm{IR}}/k_{\mathrm{V}}$ denotes the ratio of IR and visible absorption coefficients. The radiative transfer equation becomes:

$$u\frac{dI(\tau,u)}{d\tau} = I(\tau,u) - B(\tau) = I(\tau,u) - \frac{1}{2}\int_{-1}^{+1} du I(\tau,u) - \frac{F_{\mathrm{a}}^{\mathrm{s}}}{4\pi n} e^{-\tau/n\bar{\mu}_0} \qquad (51)$$

which is identical to Eq. 13: $u\frac{dI(\tau,u)}{d\tau} = I(\tau,u) - \frac{1}{2}\int_{-1}^{+1} du I(\tau,u)$ except that it contains an "imbedded source": $\frac{F_{\mathrm{a}}^{\mathrm{s}}}{4\pi n} e^{-\tau/n\bar{\mu}_0}$.

Role of Radiation in Climate: Radiative Equilibrium with Finite Visible Optical Depth (3)

This equation is mathematically identical to that of *collimated (solar) beam incidence*, for conservative ($\varpi = 1$), isotropic ($p = 1$) scattering. Thus, if we make the correspondences (see Eq. 7.89 in STS):

$$F^{\mathrm{s}} \to F_{\mathrm{a}}^{\mathrm{s}}/n \qquad \text{and} \qquad \mu_0 \to n\bar{\mu}_0$$

then the two-stream solution for the source function for a **semi-infinite** atmosphere is found to be ($m \equiv \mu_0/\bar{\mu} \to n\bar{\mu}_0/\bar{\mu} \equiv \gamma$):

$$\begin{aligned} S(\tau) = B(\tau) &= \frac{F^{\mathrm{s}}}{4\pi}[(1-m^2)e^{-\tau/\mu_0} + m(1+m)] \\ &= \frac{F_{\mathrm{a}}^{\mathrm{s}}}{4\pi n}[(1-\gamma^2)e^{-\tau/\gamma\bar{\mu}} + \gamma(1+\gamma)] \end{aligned} \tag{52}$$

where γ = ratio of the slant opacity in the IR ($\tau_{\mathrm{IR}}/\bar{\mu}$) to that in the visible ($\tau_{\mathrm{V}}/\bar{\mu}_0$):

$$\gamma \equiv n\bar{\mu}_0/\bar{\mu} = \frac{k_{\mathrm{IR}}/\bar{\mu}}{k_{\mathrm{V}}/\bar{\mu}_0} = \frac{\tau_{\mathrm{IR}}/\bar{\mu}}{\tau_{\mathrm{V}}/\bar{\mu}_0}.$$

Requiring that the absorbed solar energy be linked directly to T_{e}, we set:

$\bar{\mu}_0 F_{\mathrm{a}}^{\mathrm{s}} = 2\bar{\mu}\sigma_{\mathrm{B}} T_{\mathrm{e}}^4 \quad \longrightarrow \quad S(\tau) = B(\tau) = \sigma_{\mathrm{B}} T_{\mathrm{e}}^4/\pi$ for $\gamma = 1$ (isothermal case, $T = T_{\mathrm{e}}$) consistent with Eq. 52 for $\gamma = 1$.

Role of Radiation in Climate: Radiative Equilibrium with Finite Visible Optical Depth (4)

Using the above correspondences, and $S(\tau) = \sigma_{\rm B} T^4(\tau)/\pi$, we find that the greenhouse factor may be expressed in terms of the parameter γ as:

$$\mathcal{G}(\tau) \equiv (T(\tau)/T_{\rm e})^4 = \frac{1}{2\gamma}[(1-\gamma^2)e^{-\tau/\gamma\bar{\mu}} + \gamma(1+\gamma)]. \tag{53}$$

The temperature profiles versus optical depth for several different values of γ are shown in Fig. 23. There are three interesting cases:

- (1) $\gamma >> 1$, or equivalently $k_{\rm IR} >> k_{\rm V}$ => **the strong greenhouse limit** => solar radiation penetrates deeply in the atmosphere => resulting IR radiation is 'trapped': Atmosphere acts like a one-way "valve" allowing incoming energy $(\bar{\mu}_0 F_{\rm a}^{\rm s})$ to enter easily but the resulting IR energy to escape with difficulty.

In the deep atmosphere, the greenhouse enhancement 'saturates':

$$\mathcal{G}(\tau^* \to \infty) = (1+\gamma)/2 = (1/2 + n\bar{\mu}_0/2\bar{\mu}).$$

Thus, the asymptotic temperature is:

$$T(\tau^* \to \infty) = T_{\rm e}(1/2 + n\bar{\mu}_0/2\bar{\mu})^{1/4} \approx T_{\rm e}\Big(\frac{k_{\rm IR}/\bar{\mu}}{k_{\rm V}/\bar{\mu}_0}\Big)^{1/4}.$$

Role of Radiation in Climate: Radiative Equilibrium with Finite Visible Optical Depth (5)

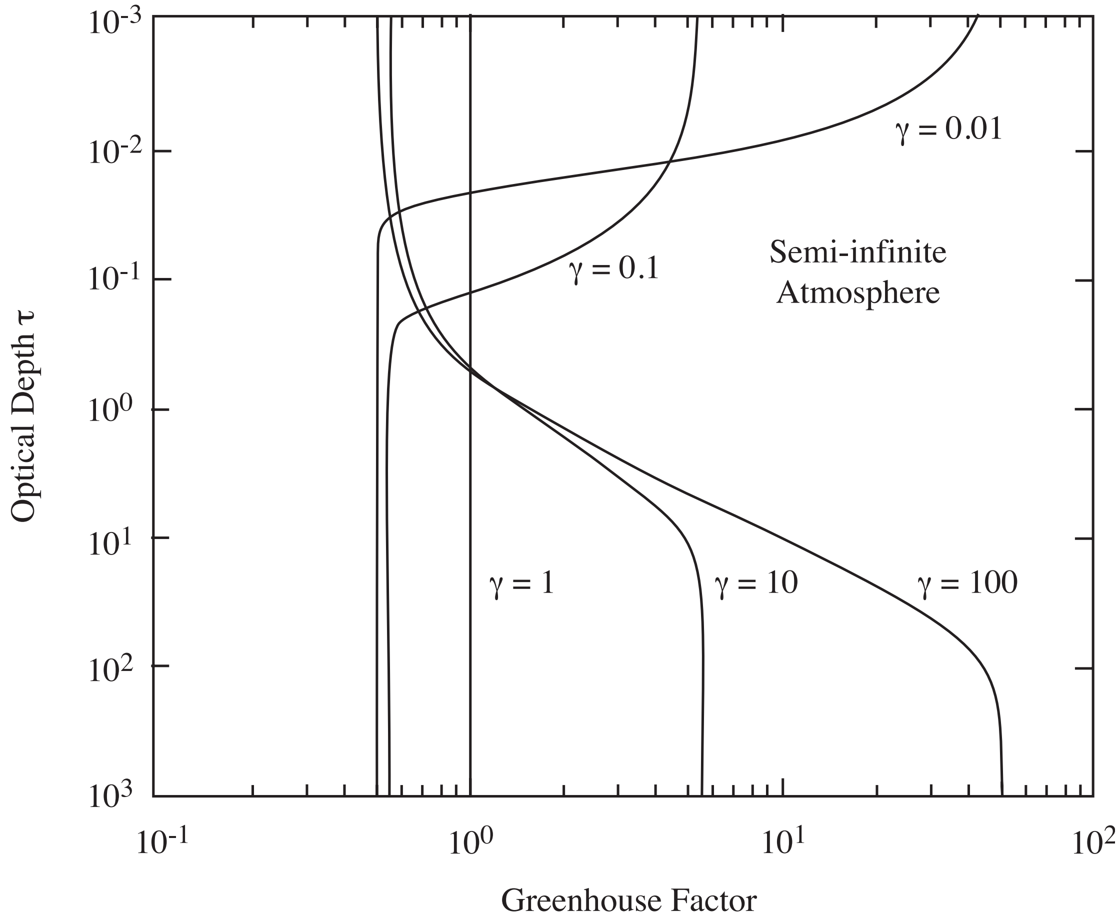


Figure 23: **Greenhouse factor $\mathcal{G}$ for a homogeneous, semi-infinite atmosphere versus optical depth for five different values of γ, the ratio of infrared slant optical depth to the visible slant optical depth.**

Role of Radiation in Climate: Radiative Equilibrium with Finite Visible Optical Depth (6)

- As expected: the two-stream approximation (TSA) to the gray radiative transfer problem is quite accurate, for this optically-thick, gray situation.
- The resemblance of the $\gamma >> 1$ solutions to the temperature structure of Venus was noted in the 1960's, when observations indicated T_s near 800 K.
- A pure radiative-equilibrium solution is found to be a good approximation for Venus' lower atmosphere. On the other hand:
- the modest greenhouse trapping experienced on Earth and Mars is not well described by Eq. 53 because of the importance of the surface in the radiative transfer, and also because of our neglect of convective heat transport.
- Next: (2) $\gamma = \frac{k_{\rm IR}/\bar{\mu}}{k_{\rm V}/\bar{\mu}_0} = \frac{\tau_{\rm IR}/\bar{\mu}}{\tau_{\rm V}/\bar{\mu}_0} = 1$: isothermal ($T(\tau) = T_{\rm e}$) situation: solar heating exactly balances IR escape.
- The $n = k_{\rm IR}/k_{\rm V} = 1$ case also describes conservative scattering from a homogeneous, semi-infinite atmosphere for which S. Chandrasekhar found an exact solution in 1949: $S(\tau \to \infty)/S(\tau = 0) = \bar{\mu}_0\sqrt{3}$: In the TSA: $\bar{\mu} = 1/\sqrt{3}$ is the "best" value to use in optically thick situations.

Role of Radiation in Climate: Radiative Equilibrium with Finite Visible Optical Depth (7)

Finally:

- (3) $\gamma << 1$, or $k_{\rm IR} << k_{\rm V}$ represents the so-called **anti-greenhouse** case.

The anti-greenhouse case is relevant to numerous phenomena in the solar system:

- An inverted temperature structure characterizes Earth's upper stratosphere, where absorption of UV radiation by ozone gives rise to a temperature inversion.

- It also describes the radiative equilibrium temperature distribution for the so-called **nuclear-winter** scenario, corresponding to a reduced surface temperature caused by a stratosphere loaded with absorbing aerosols.

- It may have occurred 65 million years ago when worldwide cooling resulted from injection of dust, as a result of an impact with a large meteoroid.

- Stratospheric aerosols (with optical depth estimated to range up to 10) resulting from the gigantic Mt. Toba volcanic eruption some $70,000$ years ago may have been responsible for a subsequent cooling of Earth's climate for a period of 200 years.

- It is also applicable to the dusty atmospheres of Mars and of Titan, a satellite of the planet Saturn.

Interesting Case Study: The Little Ice Age (1)

search and discovery

The triggering and persistence of the Little Ice Age

A mere half century of volcanism seems to have initiated a chill lasting half a millennium.

For more than 500 years until the middle of the 19th century, much of the Northern Hemisphere experienced the "Little Ice Age," the most extended period of anomalous cold—winter and summer—in 8000 years. Picturesque aspects of the LIA are familiar from paintings of winter scenes in northern Europe. But more somber manifestations include numerous famines in Europe and Asia and the extinction of the Norse settlements in southern Greenland.

The LIA's start and finish dates, as well as its cause, have long been subjects of debate and puzzlement. Variations in solar irradiation and volcanic eruptions have been invoked as possible causes. But the one seems too weak and the other too ephemeral.

GIFFORD MILLER

Figure 1. Long-dead but still rooted moss, just emerged from a receding local icecap on Canada's Baffin Island. Carbon-14 dating of such clumps provides an impressively detailed chronology of the onset of the Little Ice Age around 1300 AD and its intensification around 1450.[1]

pinpoint the LIA's abrupt onset and understand its duration.[1]

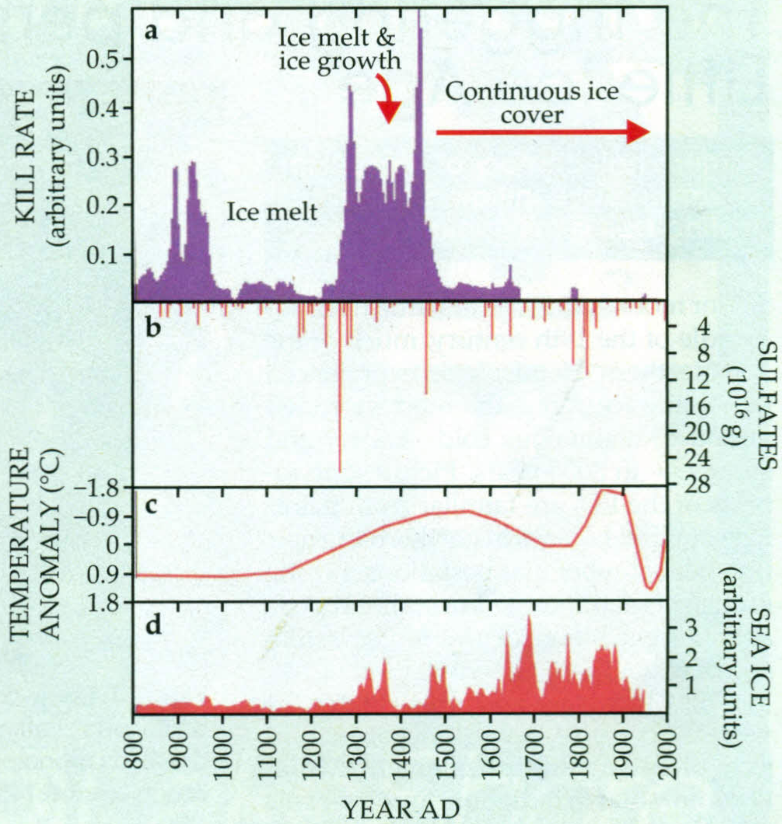
spheric load of sulfate aerosol based on dateable sulfate concentrations in Arctic

Physics Today, April 2012.

Interesting Case Study: The Little Ice Age (2)

Note: LIA cooling consistent with **anti-greenhouse effect** ($\gamma << 1$ in Eq. 53).

Figure 2. Geological data covering the last millennium. **(a)** Relative kill rates of moss by expanding local icecaps on Baffin Island, determined by carbon-14 dating. **(b)** Global mass of volcanic sulfate aerosol in the stratosphere, estimated from sulfates in ice cores. **(c)** Temperature anomalies in southern Greenland (relative to the 1880–1980 mean), determined from borehole measurements. **(d)** Concentration of ice-loving microplankton in sediment cores from the shelf off Iceland's north coast, a proxy for sea-ice accumulation there. (Adapted from ref. 1.)



Physics Today, April 2012.

Role of Radiation in Climate: Summary – Lessons learned (1)

We have learned that in the absence of an atmosphere, the effective temperature of a planet becomes:

$$T_{\rm e} = \left[\frac{S_0(1-\bar{\rho})}{4\sigma_{\rm B}}\right]^{1/4} \longleftarrow \quad \textbf{planetary RE temperature}. \tag{54}$$

- For Earth: $\bar{\rho} = 0.3$ – spherical albedo; $S_0 = 1,368$ W·m^{-2} – solar "constant."
- The above formula yields $T_{\rm e} = -18$ °C , **much lower than Earth's mean surface temperature** $T_{\rm s} = +15$ °C $\longleftarrow$ the **greenhouse effect**.

If atmosphere is transparent to visible (solar) radiation, but absorbs in the infrared:

Radiative Equilibrium Expression for Atmospheric Temperature:

$$T_{\rm re}(\tau) = T_{\rm e}\left(\frac{1}{2}+\frac{\tau}{2\bar{\mu}}\right)^{1/4} \equiv T_{\rm e}\mathcal{G}^{1/4}(\tau). \tag{55}$$

- The temperature increases monotonically downward from an outer 'skin' temperature $T_{\rm re}(0) = T_{\rm e}/(2)^{1/4}$ to $T_{\rm re}(\tau^*) = T_{\rm re}(0)(1+\frac{\tau^*}{\bar{\mu}})^{1/4} < T_{\rm s} = T_{\rm e}(1+\tau^*/2\bar{\mu})^{1/4}$ at the lower boundary.

Role of Radiation in Climate: Summary – Lessons learned (2)

If we set the greenhouse increase in temperature (over the effective temperature) equal to the product of the lapse rate $\Gamma_{\rm env}$ and $z_{\rm e}$, or:

$$T_{\rm s}(\tau^*) \cong T_{\rm e} + |\Gamma_{\rm env}| z_{\rm e} \qquad z_{\rm e} = H_{\rm a} \ln(\tau^*/\tau_{\rm e}), \tag{56}$$

we may combine the radiative-convective solution valid in the lower region (below the tropopause height, $z_{\rm t}$) and the RE solution valid above $z_{\rm t}$ (see Fig. 24):

$$T_{\rm rc}(z) = T_{\rm e} + |\Gamma_{\rm env}|[z_{\rm e} - z] \qquad (z \leq z_{\rm t}) \tag{57}$$

$$T_{\rm rc}(z) = T_{\rm e}(2)^{-1/4} \qquad (z > z_{\rm t}) \tag{58}$$

where the tropopause height is given by $z_{\rm t} = z_{\rm e} + [1 - (\frac{1}{2})^{1/4}] \times T_{\rm e}/|\Gamma_{\rm env}|$. Note:

- We have obtained a realistic mean atmosphere temperature profile, in terms of opacity τ^*, adiabatic lapse rate $|\Gamma_{\rm env}|$, and absorber scale height, $H_{\rm a}$.

Writing the total clear-sky IR optical depth as: $\tau^*_{\rm tot} = \tau^*_{\rm n} + \tau^*({\rm H_2O})$ where $\tau^*_{\rm n} = 0.788$, Eq. 56 now becomes:

$$T_{\rm s}(\tau^*_{\rm tot}) = T_{\rm e} + |\Gamma_{\rm clr}| z_e^\dagger(\tau^*_{\rm tot}) \qquad z_{\rm e}^\dagger(\tau^*_{\rm tot}) = H_{\rm clr} \ln(\{\tau^*_{\rm n} + \tau^*({\rm H_2O})\}/\tau_{\rm e}). \tag{59}$$

Role of Radiation in Climate: Summary – Lessons learned (3)

In summary:

- **the existence of a tropopause temperature minimum, $T = T_{\rm e}/(2)^{1/4}$, can be understood from purely radiative considerations,** but the tropospheric lapse rate, $|\Gamma_{\rm clr}|$, is controlled by dynamical transport effects.

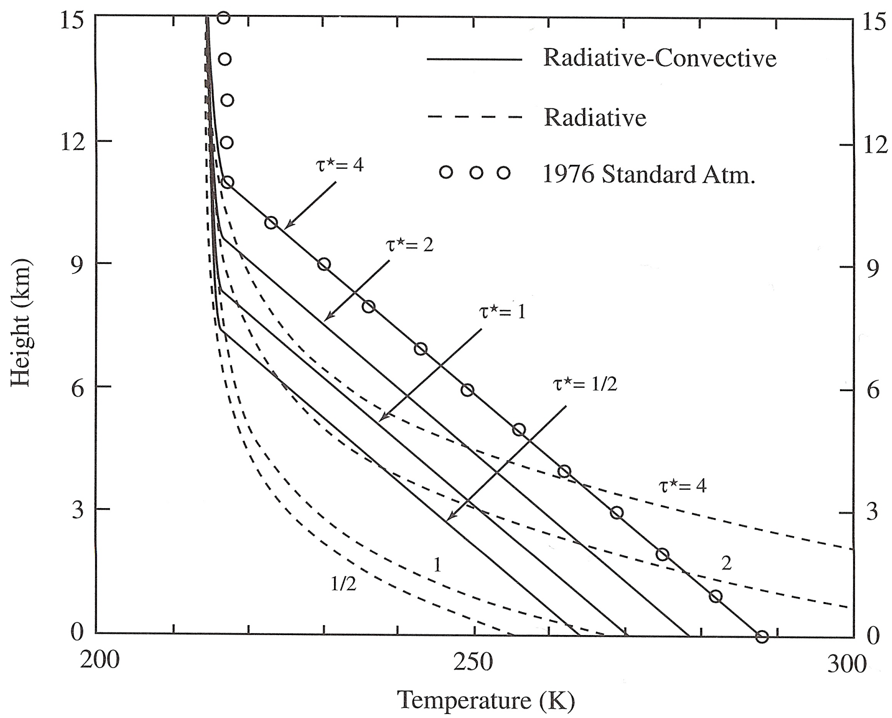


Figure 24: **Pure-radiative (dashed lines) and radiative-convective equilibrium (solid lines) temperature profiles for four different optical depths, and for $\bar{\rho} = 0.30$. Open circles: Standard Atmosphere.**

Role of Radiation in Climate: Summary – Lessons learned (4)

The surface temperature in a **cloudy** atmosphere with IR opacity $\tau_{\rm c}^*$ is:

$$
\begin{aligned}
T_{\rm s}(\tau_{\rm tot}^*) &= T_{\rm e} + |\Gamma_{\rm cld}| z_{\rm e}^{\dagger}(\tau_{\rm tot}^*) \\
z_{\rm e}^{\dagger}(\tau_{\rm tot}^*) &= H_{\rm cld} \ln[\{\tau_{\rm n}^* + \tau^*({\rm H_2O}) + \tau_{\rm c}^*\}/\tau_{\rm e}].
\end{aligned}
\tag{60}
$$

Weighting the contribution from clear air (Eq. 59) and from clouds (Eq. 60):

$$
\begin{aligned}
T_{\rm s} &= T_{\rm e} + A_{\rm c} H_{\rm cld} |\Gamma_{\rm cld}| \ln[(\tau_{\rm n}^* + \tau^*({\rm H_2O}) + \tau_{\rm c}^*)/\tau_e] \\
&+ (1 - A_{\rm c}) H_{\rm clr} |\Gamma_{\rm clr}| \ln[(\tau_{\rm n}^* + \tau^*({\rm H_2O}))/\tau_{\rm e}].
\end{aligned}
\tag{61}
$$

- Earth is partially covered with clouds with a fractional coverage $0 \le A_{\rm c} \le 1$;
- both the scale height (either $H_{\rm clr}$ or $H_{\rm cld}$) and the lapse rate (either $\Gamma_{\rm clr}$ or $\Gamma_{\rm cld}$) are allowed to vary between clear and cloudy regions;
- Eq. 61 may be used to study how the surface temperature depends on variable cloud cover $A_{\rm c}$, moisture content of the air $\tau^*({\rm H_2O})$, and IR cloud opacity $\tau_{\rm c}^*$.

Role of Radiation in Climate: Summary – Lessons learned (5)

How does the albedo of the **surface-atmosphere-cloud 'system'** affect the overall energy balance of the planet? First, recall that:

- the Earth's surface reflects visible light: average (spherical) albedo $\bar{\rho}_{\mathrm{s}} = 0.11$.
- the spherical albedo of the atmosphere (assuming a dark surface) $\bar{\rho}_{\mathrm{a}} = 0.07$.

The **spherical albedo** of the **clear-sky** atmosphere-surface system is:

$$\bar{\rho}_{\mathrm{as}} = \bar{\rho}_{\mathrm{a}} + \frac{\bar{\rho}_{\mathrm{s}}(1-\bar{\rho}_{\mathrm{a}})^2}{1-\bar{\rho}_{\mathrm{s}}\bar{\rho}_{\mathrm{a}}} = 0.166 \quad \longleftarrow \quad \textbf{clear-sky albedo}. \tag{62}$$

The **spherical albedo** of the **cloudy** atmosphere-surface system is:

$$\bar{\rho}_{\mathrm{cas}} = \bar{\rho}_{\mathrm{c}} + \frac{\bar{\rho}_{\mathrm{as}}(1-\bar{\rho}_{\mathrm{c}})^2}{1-\bar{\rho}_{\mathrm{as}}\bar{\rho}_{\mathrm{c}}} \quad \longleftarrow \quad \textbf{cloudy-sky albedo}. \tag{63}$$

- Weighting the clear-sky albedo $\bar{\rho}_{\mathrm{as}}$ with the fraction of clear sky $(1 - A_{\mathrm{c}})$, we approximate the total spherical albedo of the Earth system by:

$$\bar{\rho}_{\mathrm{tot}} = A_{\mathrm{c}}\bar{\rho}_{\mathrm{cas}} + (1-A_{\mathrm{c}})\bar{\rho}_{\mathrm{as}} \quad \longleftarrow \quad \textbf{total albedo}. \tag{64}$$

Role of Radiation in Climate: Summary – Lessons learned (6)

The cloud spherical albedo is:

$$\bar{\rho}_{\mathrm{c}} = \frac{2b_{\mathrm{c}}\tau_c^*}{2b_{\mathrm{c}}\tau_{\mathrm{c}}^* + 2\bar{\mu}} \quad \longleftarrow \quad \textbf{spherical cloud albedo} \tag{65}$$

where cloud optical depth τ_{c}^* and backscattering ratio b_{c} depend on amount of water (LWP) in the cloud and cloud particle size.

If we define a critical albedo (referenced to a clear atmosphere with albedo $\bar{\rho}_{\mathrm{as}}$):

$$\bar{\rho}_{\mathrm{crit}} = \frac{\bar{\rho}_{\mathrm{as}} + a\,\mathrm{LWP}}{1 + a\,\mathrm{LWP}}; \qquad a = 2.622 \times 10^{-3} \tag{66}$$

then clouds will result in:

- a net **cooling** if $\bar{\rho}_{\mathrm{tot}} > \bar{\rho}_{\mathrm{crit}}$ BUT
- a net **warming**, if $\bar{\rho}_{\mathrm{tot}} < \bar{\rho}_{\mathrm{crit}}$

as illustrated in Fig. 17 (reproduced below).

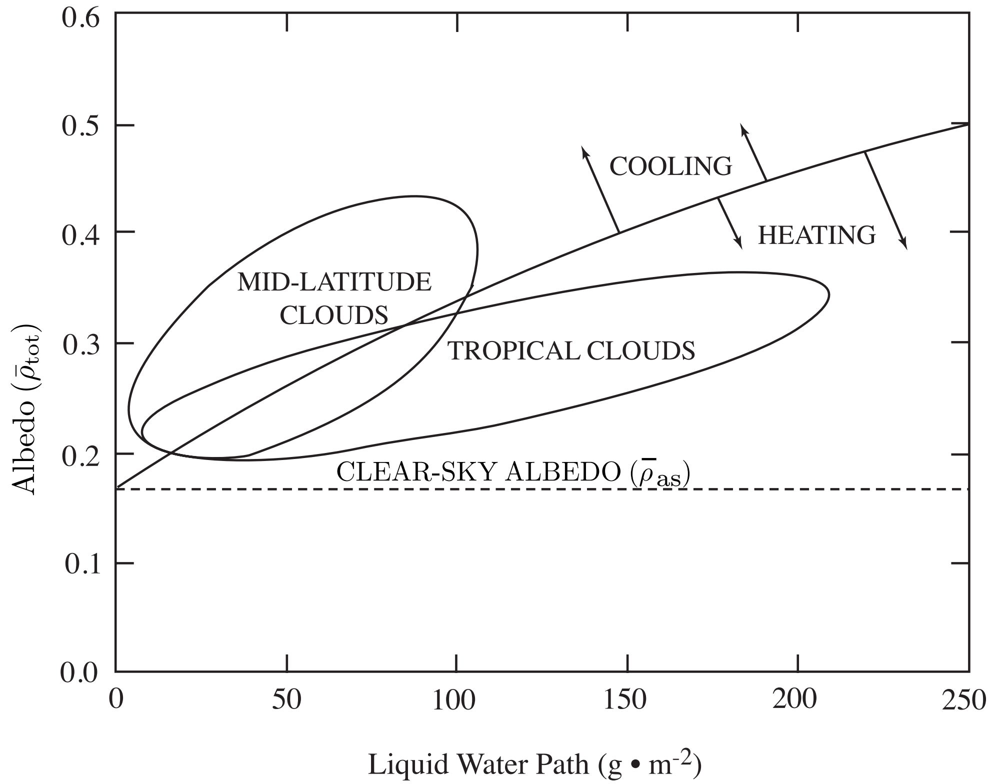


Figure 25: **Albedo versus liquid water path of clouds. Heavy solid line: Critical planetary albedo $\bar{\rho}_{crit}$ dividing regions of net warming and net cooling, given by Eq. 38. The albedo of the clear-air plus surface $\bar{\rho}_{as}$ is set equal to 0.166, as inferred from Eq. 34. The ellipses define the regions containing 97% of all observations for the tropics and midlatitudes. They were computed from Eq. 35 where $\bar{\rho}_c$ is given by the ellipses in Fig. 16. If $\bar{\rho}_{tot}$ is in the upper domain ($\bar{\rho}_{tot} > \bar{\rho}_{crit}$), net cooling occurs. If it is in the lower domain, ($\bar{\rho}_{tot} < \bar{\rho}_{crit}$), net warming occurs.**

Role of Radiation in Climate: Summary – Lessons learned (8)

How will shortwave absorption (ignored above) affect the mean radiative balance? Defining γ as the ratio of the IR slant opacity ($\tau_{\rm IR}/\bar{\mu}$) to that in the visible ($\tau_{\rm V}/\bar{\mu}_0$):

$$\gamma \equiv n\bar{\mu}_0/\bar{\mu} = \frac{k_{\rm IR}/\bar{\mu}}{k_{\rm V}/\bar{\mu}_0} = \frac{\tau_{\rm IR}/\bar{\mu}}{\tau_{\rm V}/\bar{\mu}_0}$$

we find that the greenhouse factor may be expressed in terms of γ as:

$$\mathcal{G}(\tau) \equiv (T(\tau)/T_{\rm e})^4 = \frac{1}{2\gamma}[(1-\gamma^2)e^{-\tau/\gamma\bar{\mu}} + \gamma(1+\gamma)]. \tag{67}$$

There are three interesting cases:

- (1) $\gamma >> 1$, or equivalently $k_{\rm IR} >> k_{\rm V} \Longrightarrow$ **the strong greenhouse limit** $\Longrightarrow$ Atmosphere acts like a one-way "valve" allowing incoming solar energy to enter easily but the resulting IR energy to escape with difficulty.
- (2) $\gamma = \frac{k_{\rm IR}/\bar{\mu}}{k_{\rm V}/\bar{\mu}_0} = \frac{\tau_{\rm IR}/\bar{\mu}}{\tau_{\rm V}/\bar{\mu}_0} = 1$: isothermal ($T(\tau) = T_{\rm e}$) situation: solar heating exactly balances IR escape.
- (3) $\gamma << 1$, or $k_{\rm IR} << k_{\rm V}$ represents the so-called **anti-greenhouse** case: Earth's upper stratosphere, volcanic eruptions (the **Little Ice Age**), nuclear winter, Mars, Titan.

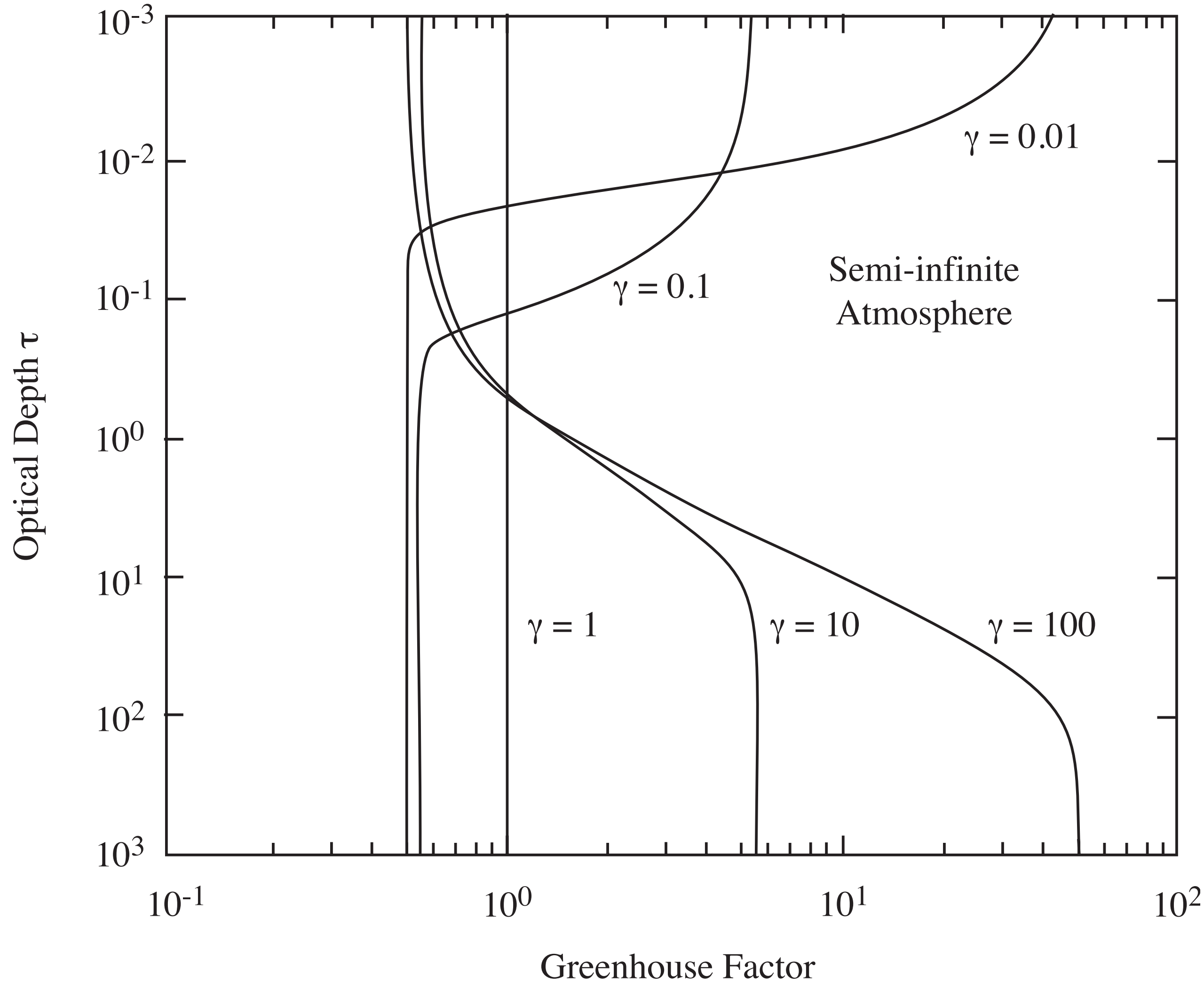


Figure 26: **Greenhouse factor $\mathcal{G}$ for a homogeneous, semi-infinite atmosphere versus optical depth for five different values of γ, the ratio of infrared slant optical depth to the visible slant optical depth.**

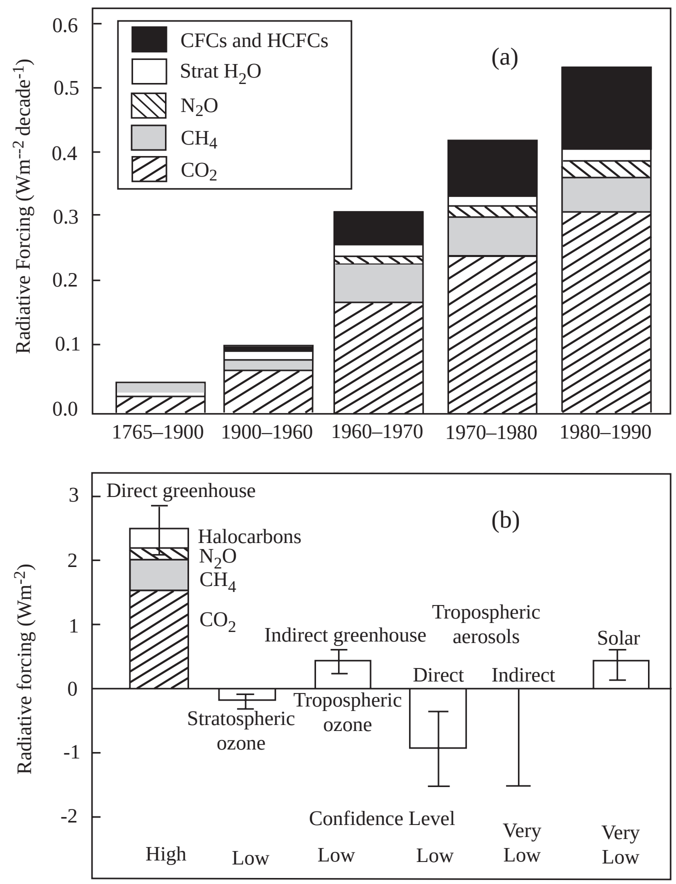


Figure 27: **(a) The degree of radiative forcing produced by selected greenhouse gases in five different epochs. Until about 1960, nearly all the forcing was due to CO_2; today the other greenhouse gases combined nearly equal the CO_2 forcing. (b) Estimates of the globally averaged radiative forcing due to changes in greenhouse gases and aerosols from pre-industrial times to the present day and changes in solar variability from 1850 to the present day.**